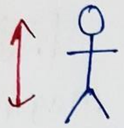
[redacted] | Class 12 maths | Vector Algebra

[सदिश बीजगणित]

Basic Concepts (Part - I)

Introduction | Types of Vectors

Introduction :



Q: What is your Height?

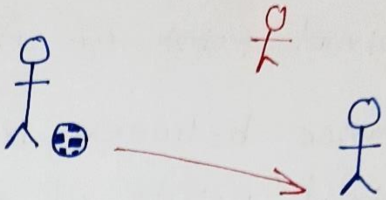
A: 178 cm

$$\boxed{\text{Magnitude only}}$$

↓

Scalar / अदिश

e.g. length, mass, time, distance, speed, area, volume, temperature, work, money, voltage, density, resistance....



Q. How will you pass football to your teammate?

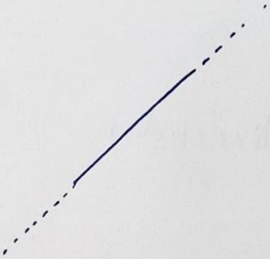
A: I will kick the football with sufficient muscle power towards my teammate.

$$\boxed{\text{Magnitude}} + \boxed{\text{Direction}}$$

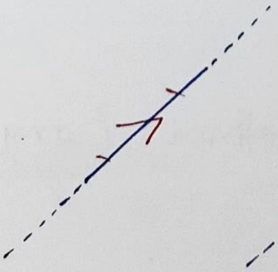
↓

Vector / सदिशं

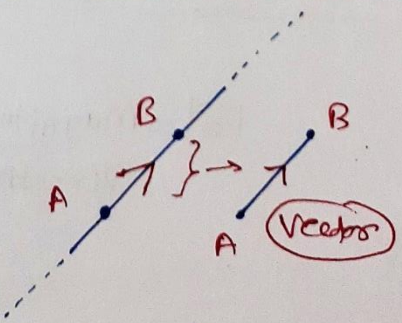
e.g. displacement, velocity, $\boxed{F=mg}$ acceleration, force, weight, momentum, electric field intensity etc....



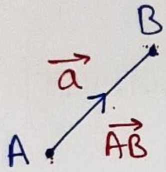
Line



Directed line



Directed line Segment



= Directed line Segment AB = Vector AB = Vector a

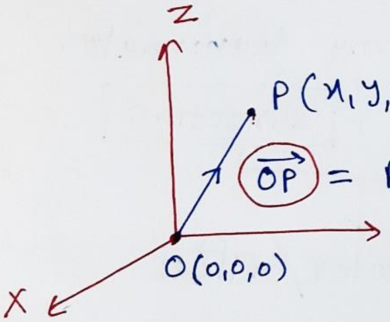
($\vec{AB}$) ($\vec{a}$)

- Arrow = Direction
- Initial Point of $\vec{AB}$ = A
- Terminal Point of $\vec{AB}$ = B
- Distance between initial and terminal points of a vector = Magnitude (length) of the vector

$|\vec{AB}| = |\vec{a}|$

$\vec{a} = (\vec{AB})$

Position Vector of a Point (स्थिति सदिश)



$\boxed{\vec{OP}}$ = Position Vector of Point P (with respect to O always)

↓ origin

$$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$$

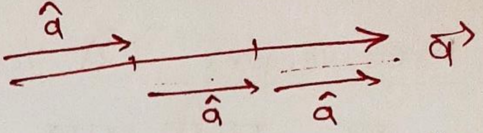
Types of Vectors →

Zero Vector (Null Vector) ($\vec{0}$) = Same initial & terminal points.

$|\vec{0}|$ = Magnitude = 0

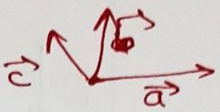
Direction = Not Defined (any direction)

Unit Vector : magnitude = 1 unit

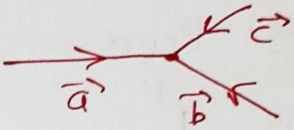


(The unit vector in the direction of $\vec{a}$ is denoted by $\hat{a}$)

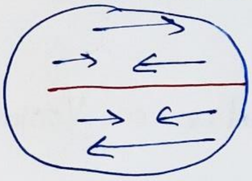
Coinitial Vectors : Two or more vectors with same initial point.



Coterminal Vectors : Two or more vectors with same terminal point.

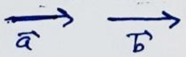


Collinear Vectors These are parallel to the same line.



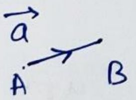
[Magnitude = Same or different
Direction = Same or opposite]

Equal Vectors. $\vec{a} = \vec{b}$ if Magnitude = Same, direction = Same.

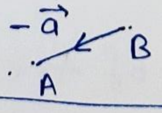


Negative of a Vector Magnitude = Same, Direction = opposite

Vector $\vec{a} = \vec{AB}$

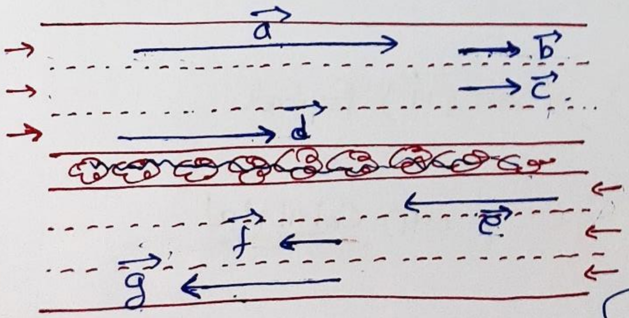


Negative vector $-\vec{a} = \vec{BA}$



$\boxed{\vec{BA} = -\vec{AB}}$

Highway



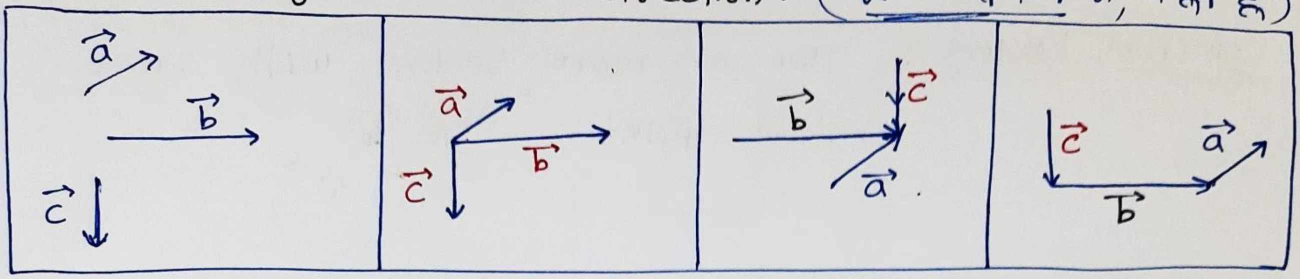
Collinear Vectors → $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$

Equal Vectors $\vec{b}, \vec{c}$ $\vec{e}, \vec{g}$

Negative of each other $\vec{e} = -\vec{d}$

$\vec{b} = -\vec{f}$, $\vec{c} = -\vec{f}$, $\vec{g} = -\vec{d}$

Free Vectors ऐसे Vectors जिनको हम एक जगह से उठा कर दूसरी जगह shift कर सकते है without changing its magnitude and direction. (पूरे chapter में, ये ही है)

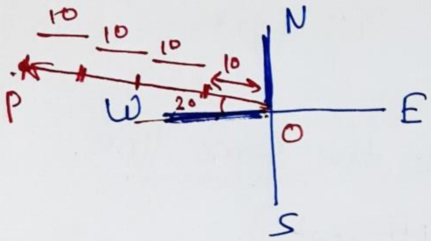


Example. Represent graphically a displacement of 40 km, 20° north of west.

vector

$\overrightarrow{OP}$ //

N W



Example. Classify the following measures as Scalar or Vector.

10 seconds → Scalar

5 g/cm^3 → Scalar

15 Newton → Vector

7 m/s^2 → Vector

acc.

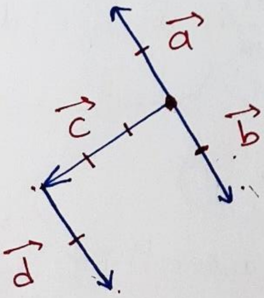
100 watt → Scalar

90° → Scalar

30 km/hr → Scalar

30 km/hr towards North → vector

Example. which of the following vectors are



$\vec{a}, \vec{b}, \vec{d}$ ← (i) Collinear

$\vec{b} = \vec{d}$ ← (ii) Equal

$\vec{a}, \vec{b}, \vec{c}$ ← (iii) Coinitial

Exercise 10.1

Q.1 Represent graphically a displacement of 40 km, 30° east of north.

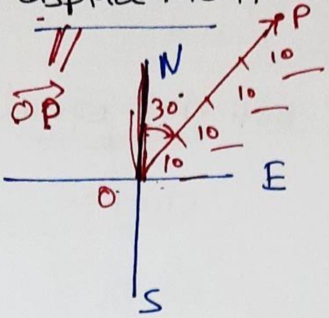
Vector

10 × 4

$\overrightarrow{OP}$

length

$|\overrightarrow{OP}|$ = magnitude



Q.2 Classify the following as scalars and vectors.

(i) 10 kg = mass = Scalar

(ii) 2 meters north-west = Displacement = vector

(iii) 40° = Scalar

(iv) 40 watt = power = Scalar

(v) 10^{-19} coulomb = Charge = Scalar

(vi) 20 m/s^2 = acceleration = vector

Q.3 Classify the following as scalars and Vectors.

(i) time period = Scalar

(ii) distance = Scalar

(iii) force = vector

(iv) velocity = vector

(v) work done = Scalar

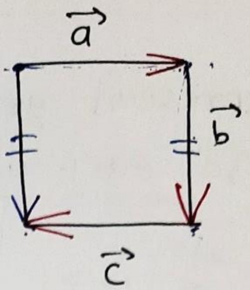
Q.4 In Fig. (a square), identify the following vectors -

(i) coinitial $\vec{a}$ & $\vec{d}$

(ii) equal = $\vec{b} = \vec{d}$

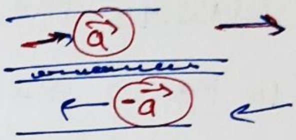
(iii) collinear but not equal $\vec{a}, \vec{c}$

∥ ↓ not in same direction



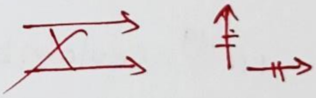
Q.5 Answer the following as true or false:

(i) $\vec{a}$ and $-\vec{a}$ are collinear. → T

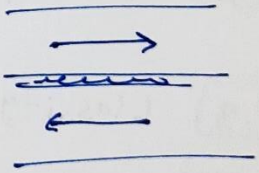


(ii) Two collinear vectors are always equal in magnitude. → F

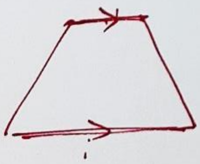
(iii) Two vectors having same magnitude are collinear. → F



(iv) Two collinear vectors having the same magnitude are equal → F



Same Dirⁿ.
Same mag.



[redacted]	Class 12 Maths	Vector Algebra

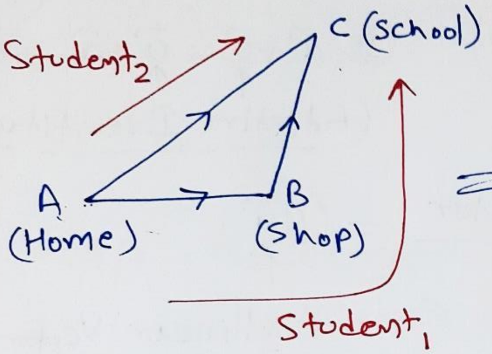
(सदिश बीजगणित)

Basic Concepts (Part-II)

Distance (दूरी) = Scalar

Displacement (विस्थापन) = Vector

Addition of Vectors



⇒ दोनों की Distance = Different

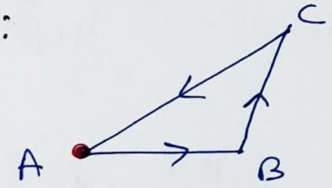
दोनों का Displacement = Same

vector

☆ $\boxed{\underbrace{\overrightarrow{AB}+\overrightarrow{BC}}_{①} = \underbrace{\overrightarrow{AC}}_{②}}$

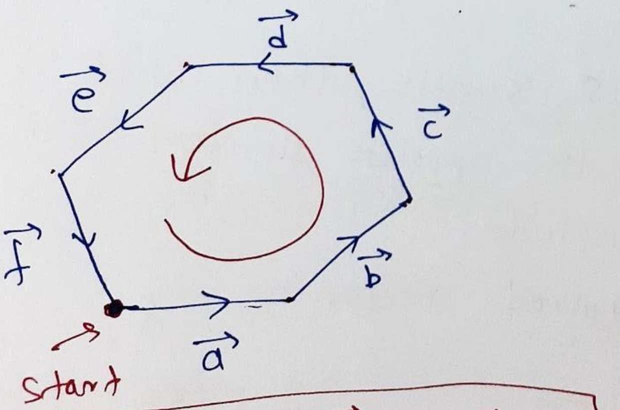
Triangle law of Addition

Note:



$\boxed{\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}} = \overrightarrow{AA}$

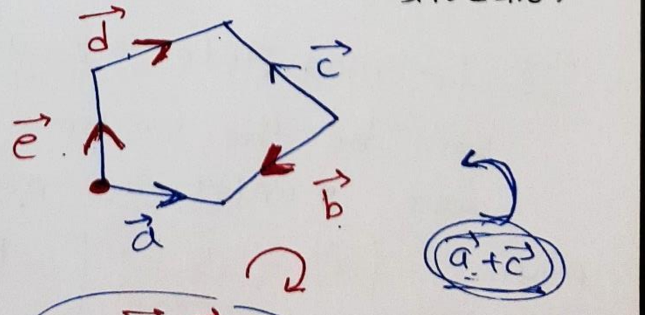
Note: In a closed circuit, Sum of all vectors in the same direction $= \vec{0}$



$\boxed{\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}=\vec{0}}$

In a circuit,

sum of vectors in one direction = Sum of vectors in another Direction

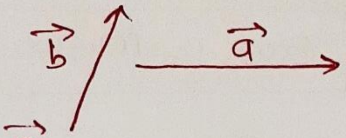


$\vec{a}+\vec{c}$

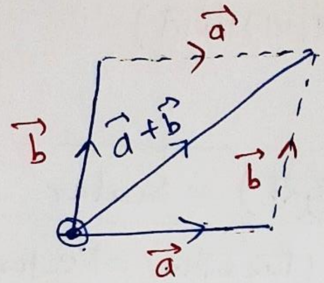
$\vec{e}+\vec{d}+\vec{b}$

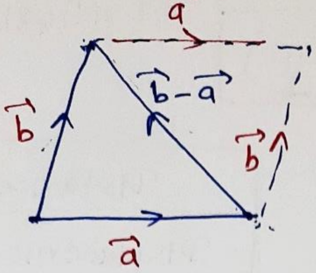
$\boxed{\vec{a}+\vec{c}=\vec{e}+\vec{d}+\vec{b}}$

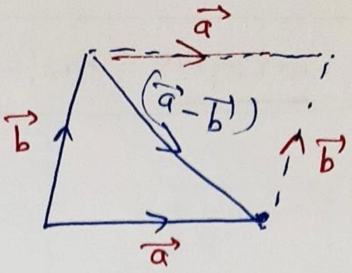
Parallelogram Law :



(Free Vectors)







Properties of Vector Addition ① $\vec{a}+\vec{b}=\vec{b}+\vec{a}$

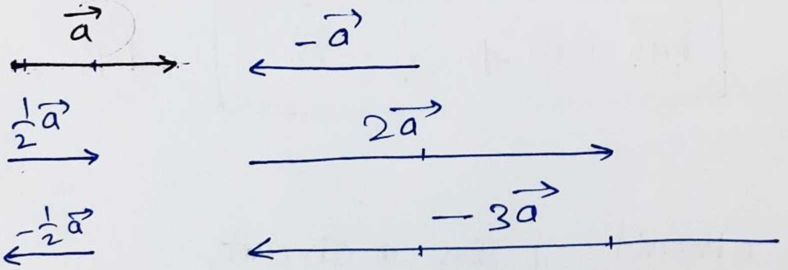
② $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ ③ $\vec{a}+\underset{\uparrow}{\vec{0}}=\underset{\uparrow}{\vec{0}}+\vec{a}=\vec{a}$

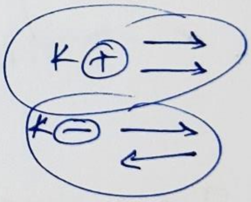
(Additive Identity)

Scalar Multiplication with Vector $= \boxed{k\vec{a}}$

$\downarrow$ k $\downarrow$ $\boxed{\vec{a}}$ ← (Collinear Vectors)

Parallel < $\vec{a} \Rightarrow$ length $|\vec{a}|$; $k\vec{a} \Rightarrow$ length $|k\vec{a}|=|k||\vec{a}|$ > length Different





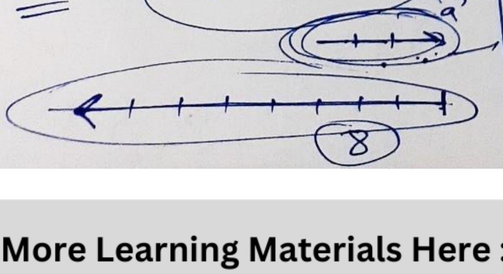
Note. Unit vector in the direction of $\vec{a} = \hat{a} = \frac{\vec{a}}{|\vec{a}|}$

e.g. If magnitude of $\vec{a}$ is 3 units, then what will be the vector in the opposite direction of $\vec{a}$ and 8 units as magnitude.

Ans. $|\vec{a}| = 3$

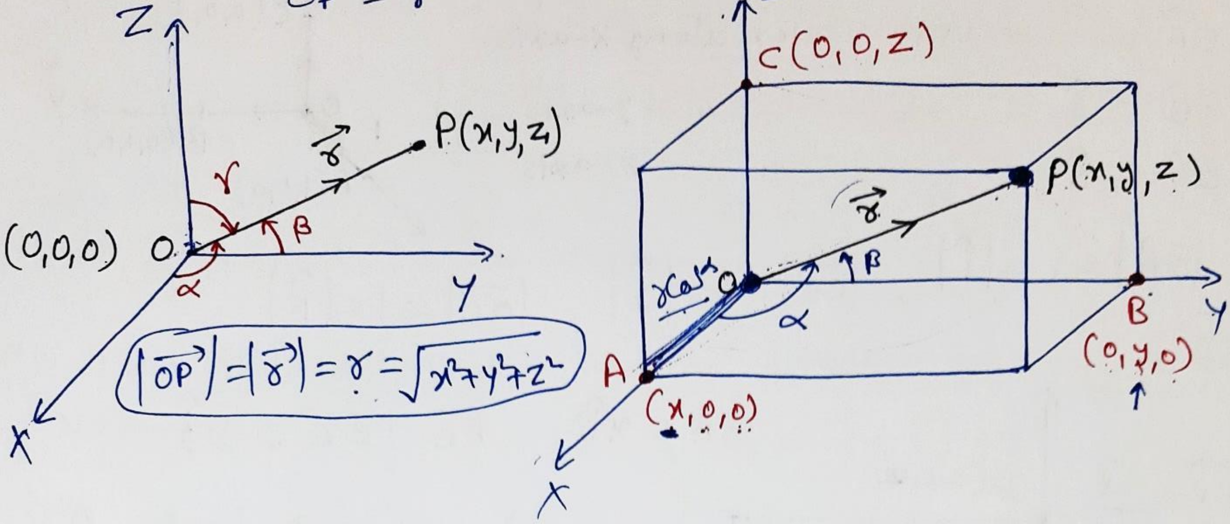
Required vector $= -8\frac{\vec{a}}{3}$

$= -\frac{8}{3}\vec{a}$



Direction Cosines, Direction Ratios →

$\vec{OP} = \vec{r}$



$$\boxed{|\vec{OP}| = |\vec{r}| = r = \sqrt{x^2+y^2+z^2}}$$

☆ $\alpha, \beta, \gamma \rightarrow$ Direction angles between $\vec{r}$ and Positive sides of X, Y, Z-axis respectively.

☆ $\underset{\downarrow \atop l}{\cos\alpha}, \underset{\downarrow \atop m}{\cos\beta}, \underset{\downarrow \atop n}{\cos\gamma} \rightarrow$ Direction Cosines

☆ $\lambda\cos\alpha, \lambda\cos\beta, \lambda\cos\gamma \rightarrow$ Direction Ratios (Proportional to D.C.)

$\boxed{\lambda \neq 0}$ $(\lambda l, \lambda m, \lambda n)$

By the Diagram.

Point (x, y, z)

$x = r\cos\alpha \rightarrow \cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}$ (D.C.)

$y = r\cos\beta \rightarrow \cos\beta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2+z^2}}$

$z = r\cos\gamma \rightarrow \cos\gamma = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

Note: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

or

$l^2 + m^2 + n^2 = 1$

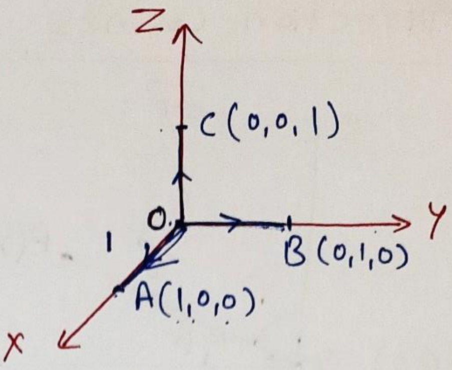
Note: Direction Ratios.

$\rightarrow \lambda\cos\alpha, \lambda\cos\beta, \lambda\cos\gamma$

$\rightarrow r\cos\alpha, r\cos\beta, r\cos\gamma$ $\boxed{\lambda = r}$

$\rightarrow \boxed{x, y, z}$ ✓

Components of a Vector :→

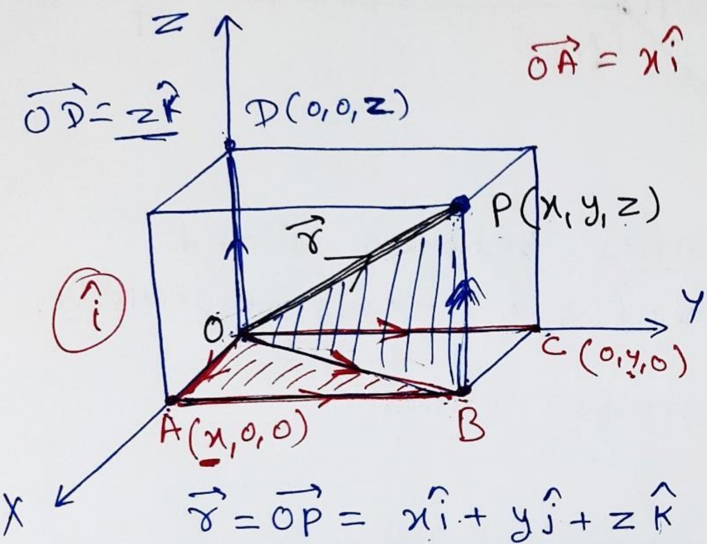


$\vec{OA} = \hat{i}$ = unit vector along x-axis

$\vec{OB} = \hat{j}$ = ———— " — y-axis

$\vec{OC} = \hat{k}$ = ———— " —— z-axis

$|\vec{OA}| = 1 = |\hat{i}|$, $|\vec{OB}| = 1 = |\hat{j}|$, $|\vec{OC}| = 1 = |\hat{k}|$



$\vec{OA} = x\hat{i}$ $\vec{AB} = \vec{OC} = y\hat{j}$

$\vec{OB} = \vec{OA} + \vec{AB} = x\hat{i} + y\hat{j}$

$$\vec{r} = \vec{OP} = \vec{OB} + \vec{BP}$$
$$= x\hat{i} + y\hat{j} + z\hat{k}$$

$\vec{r} = \vec{OP} = \underbrace{x\hat{i} + y\hat{j} + z\hat{k}}_{\text{Component Form}}$ = position vector of P(x, y, z)

Vector Components
$x\hat{i}$, $y\hat{j}$, $z\hat{k}$

Scalar Components
x, y, z

length OP = $|\vec{OP}| = |\vec{r}| = |x\hat{i} + y\hat{j} + z\hat{k}| = \sqrt{x^2 + y^2 + z^2}$

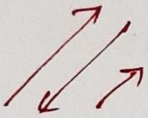
Note: $(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) \pm (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) = (x_1 \pm x_2)\hat{i} + (y_1 \pm y_2)\hat{j} + (z_1 \pm z_2)\hat{k}$

Note: Equal Vectors $\vec{a} = \vec{b}$

$$x_1\hat{i} + y_1\hat{j} + z_1\hat{k} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$\boxed{x_1 = x_2}$ $\boxed{y_1 = y_2}$ $\boxed{z_1 = z_2}$

Note : for Collinear Vectors. $\vec{a} = \lambda \cdot \vec{b}$



$$\Rightarrow (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) = \lambda(x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$

$\boxed{x_1 = \lambda x_2}$ $\boxed{y_1 = \lambda y_2}$ $\boxed{z_1 = \lambda z_2}$

Ratio equal ⇒ Proportional $\boxed{\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = \lambda}$

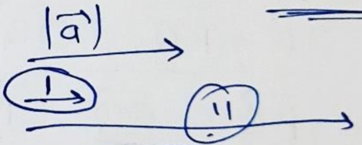
e.g. Find the unit vector in the direction of sum of the vectors, $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ & $\vec{b} = \hat{i} + 4\hat{j} + 2\hat{k}$.

Ans. $\vec{a} + \vec{b} = 3\hat{i} + 5\hat{j} - \hat{k} = \vec{c}$

$$|\vec{c}| = \sqrt{3^2 + 5^2 + (-1)^2}$$
$$= \sqrt{9 + 25 + 1}$$
$$= \sqrt{35}$$

$$\boxed{\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + 5\hat{j} - \hat{k}}{\sqrt{35}}}$$

e.g. Find a vector in the direction of vector $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ that has magnitude 11 units



$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}} \leftarrow \text{length} = 1$$

Required Vector $= 11\left(\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}\right) = \frac{11}{\sqrt{6}}(\hat{i} + 2\hat{j} + \hat{k})$

e.g. Write the direction ratio's of the vector $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$. Also find direction Cosines.

$\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$

Direction Ratio (DR) = (3, -1, 2)

✓ (6, -2, 4)

✓ 300, -100, 200

D.C.

$\sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$

$\cos\alpha = \frac{x}{\sqrt{14}} = 3/\sqrt{14}$ ✓

$\cos\beta = \frac{y}{\sqrt{14}} = -1/\sqrt{14}$ ✓

$\cos\gamma = z/\sqrt{14} = 2/\sqrt{14}$ ✓

e.g. Find the values of a & b if vectors $2\hat{i}+a\hat{j}+3\hat{k}$ and $-\hat{i}+2\hat{j}+b\hat{k}$ are collinear.

//

Condn. $\boxed{\frac{2}{-1}} = \frac{a}{2} = \frac{3}{b}$

$\Rightarrow -2 = \frac{a}{2} = \frac{3}{b}$

$\boxed{a=-4}$ ✓

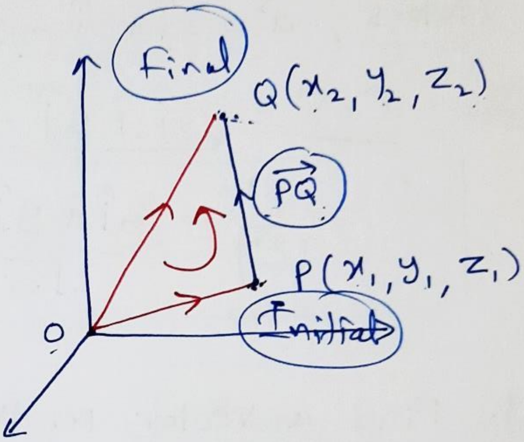
$-2 = \frac{3}{b}$

$\boxed{b = \frac{3}{-2}}$ ✓

★Vector Joining Two Points

$\vec{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{OQ} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



By Triangle law of Addition.

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\Rightarrow \boxed{\vec{PQ} = \vec{OQ} - \vec{OP}} = $ Final − Initial

$\Rightarrow \vec{PQ} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$

$\Rightarrow \boxed{\vec{PQ} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}$

$|\vec{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$ ✓

e.g. Find the vector Joining A(1,2,3) and B(3,2,4) directed from A to B.

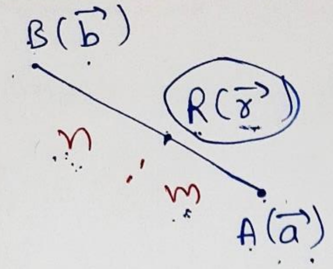
Ans $\vec{AB} = (2)\hat{i} + (0)\hat{j} + (4-3)\hat{k}$

initial Final

$\boxed{\vec{AB} = 2\hat{i} + \hat{k}}$

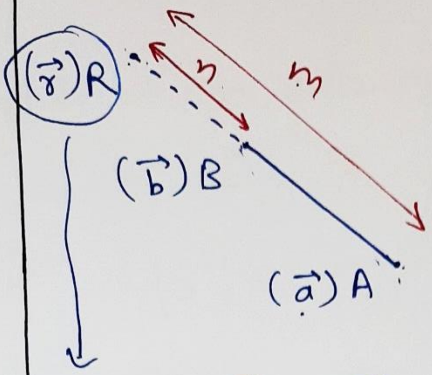
Section Formula

Internal Division



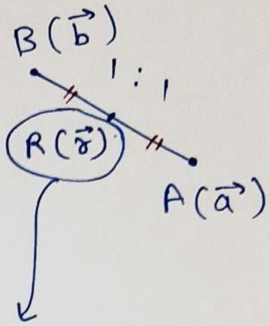
$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

External Division



$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

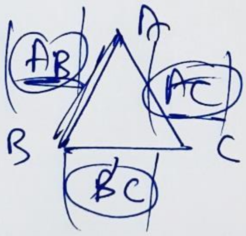
Mid Point Formula



$$\vec{r} = \frac{\vec{a} + \vec{b}}{2}$$

---

e.g. Show that the points $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$, $C(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right angled triangle.



Pythagoras Theorem

Final - initial

$$\vec{AB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{CA} = (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k}) = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$|\vec{AB}| = \sqrt{1+4+36} = \sqrt{41} = AB$$

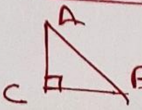
$$|\vec{BC}| = \sqrt{4+1+1} = \sqrt{6} = BC$$

$$|\vec{CA}| = \sqrt{1+9+25} = \sqrt{35} = CA.$$

$41 = 35 + 6$

$\Rightarrow (\sqrt{41})^2 = (\sqrt{35})^2 + (\sqrt{6})^2$

$\Rightarrow \boxed{AB^2 = (CA)^2 + BC^2}$



Exercise 10.2

Q.1 Compute the magnitude of the following vectors :

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}-7\hat{j}-3\hat{k}$, $\vec{c} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$.

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k} \Rightarrow |\vec{r}| = \sqrt{x^2+y^2+z^2}$ ✓

$|\vec{a}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$,

$|\vec{c}| = \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= \sqrt{1} = 1$

$|\vec{b}| = \sqrt{(2)^2+(-7)^2+(-3)^2}$

$= \sqrt{4+49+9}$

$= \sqrt{62}$

Q.2 Write two ~~to~~ different vectors having same magnitude.

$\hat{i}, \hat{j}$ $\quad$ $\hat{i}+\hat{j}+2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$

Q.3 Write two different vectors having same direction.

$\hat{i}, 2\hat{i}$ $\quad$ $\vec{a}$ $\quad$ $k\vec{a}$ $\quad$ $(\hat{i}+\hat{j}+2\hat{k})$, $100\hat{i}+100\hat{j}+200\hat{k}$

$(+)$

Q.4 Find the values of x & y so that vectors $\underset{\vec{a}}{2\hat{i}+3\hat{j}}$ and $\underset{\vec{b}}{x\hat{i}+y\hat{j}}$ are equal.

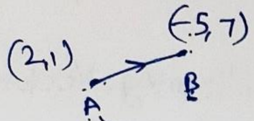
$\boxed{\vec{a} = \vec{b}}$

$\boxed{2\hat{i}+3\hat{j} = x\hat{i}+y\hat{j}}$

$\boxed{2 = x}$ $\quad$ $\boxed{3 = y}$

Q.5 Find the Scalar and vector components of the vector with initial point (2,1) and terminal point (-5,7).

Ⓐ Ⓑ



$\vec{b} = -5\hat{i}+7\hat{j}$

$\vec{AB}$ = Final - intial $\rightarrow$ $\vec{a} = 2\hat{i}+\hat{j}$

$= \vec{b}-\vec{a} = (-5\hat{i}+7\hat{j}) - (2\hat{i}+\hat{j})$

$= -7\hat{i}+6\hat{j}$

Vector Form

Vector Component.
$-7\hat{i}, 6\hat{j}$

Scalar Comp.
$-7, 6$

Q.6 Find the sum of the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b} = -2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

$$\vec{a}+\vec{b}+\vec{c} = (\cancel{1}-\cancel{2}+\cancel{1})\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k}$$

$$= 0.\hat{i} + (-4\hat{j})+(-\hat{k}) = -4\hat{j}-\hat{k}$$

Q.7 Find the unit vector in the direction of vector $\vec{a} = \hat{i}+\hat{j}+2\hat{k}$.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$$

$$|\vec{a}| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$$

$$= \frac{\hat{i}}{\sqrt{6}}+\frac{\hat{j}}{\sqrt{6}}+\frac{2\hat{k}}{\sqrt{6}}$$

Q.8 Find the unit vector in the direction of $\vec{PQ}$, where P & Q points are (1,2,3) and (4,5,6) respectively.

Ans. $\vec{r} = \vec{PQ} = (\underset{Ⓠ}{\text{Final}} - \underset{Ⓟ}{\text{initial}}) = (4-1)\hat{i}+(5-2)\hat{j}+(6-3)\hat{k}$

$$= 3\hat{i}+3\hat{j}+3\hat{k}$$

$$|\vec{r}| = |\vec{PQ}| = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}} = \frac{\hat{i}}{\sqrt{3}}+\frac{\hat{j}}{\sqrt{3}}+\frac{\hat{k}}{\sqrt{3}}$$

Q.9 For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Ans. $\vec{a} + \vec{b} = (2-1)\hat{i} + (\cancel{-1}+\cancel{1})\hat{j} + (2-1)\hat{k} = \hat{i} + \hat{k} = \boxed{\vec{c}}$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$

$$\boxed{|\vec{c}| = \sqrt{1^2 + 1^2} = \sqrt{2}}$$

Q.10 Find the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

$\vec{a}$

$$|\vec{a}| = \sqrt{25 + 1 + 4}$$
$$|\vec{a}| = \sqrt{30}$$

Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$\hat{a} = \dfrac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$ = unit vector in the Direction of $\vec{a}$

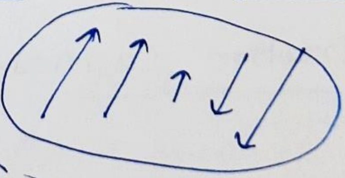
vector in the direction of $\vec{a}$ & which has magnitude 8 units $= 8(\hat{a}) = \dfrac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

Q.11 Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Proportional

↓

Ratio = Equal



Condition of Collinear Vectors

$$\frac{2}{-4} = \frac{-3}{6} = \frac{4}{-8}$$

$$\Rightarrow \boxed{-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}}$$

Q.12 Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}$.

D.C.

$$|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$$
$$= \sqrt{1+4+9}$$
$$= \sqrt{14}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$
$$\cos\beta = y/r$$
$$\cos\gamma = z/r$$

$$\cos\alpha,\ \cos\beta,\ \cos\gamma$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$\frac{1}{\sqrt{14}},\ \frac{2}{\sqrt{14}},\ \frac{3}{\sqrt{14}}$$

Q.13 Find the direction ~~ratio~~ cosines of the vector ? joining the points A(1, 2, -3) and B(-1, -2, 1), directed from A to B.

$$\underset{A}{(1,2,-3)} \longrightarrow \underset{B}{(-1,-2,1)}$$

Final - initial

initial

Final

$$\vec{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1+3)\hat{k}$$
$$\vec{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

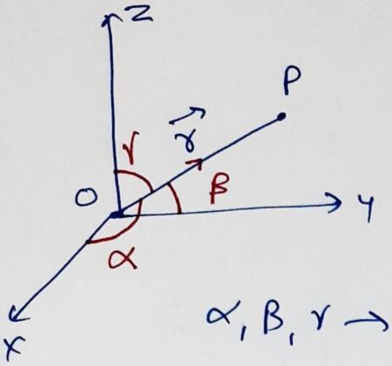
$$|\vec{AB}| = \sqrt{x^2+y^2+z^2} = \sqrt{4+16+16} = 6$$

Direction Cosines.

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{-2}{6} = -\frac{1}{3}$$
$$\cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}} = \frac{-4}{6} = -\frac{2}{3}$$
$$\cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{4}{6} = \frac{2}{3}$$

Q.14 Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY, OZ.

Same angle



$\vec{r} = \vec{OP} = \hat{i}+\hat{j}+\hat{k}$

$|\vec{r}| = \sqrt{1^2+1^2+1^2}$

$= \sqrt{3}$

To Prove,

$\alpha = \beta = \gamma$

$\alpha, \beta, \gamma \rightarrow$ Direction angles between $\vec{r}$ and positive sides of X, Y, & Z-axis.

OX OY OZ

Direction Cosines.

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{|\vec{r}|} = \frac{1}{\sqrt{3}}$$

$$\cos\beta = \frac{y}{|\vec{r}|} = \frac{1}{\sqrt{3}}$$

$$\cos\gamma = \frac{z}{|\vec{r}|} = \frac{1}{\sqrt{3}}$$

$\cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \boxed{\alpha = \beta = \gamma}$

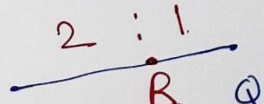
~~Ans~~

$\therefore \vec{r} = \hat{i}+\hat{j}+\hat{k}$ is equally inclined with OX, OY & OZ.

Q.15 ~~Sho~~ Find the position vectors of a point R which divides the line joining two points P & Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively in the 2 : 1 (i) internally (ii) externally.

Section Formula

(i)



$(\vec{r})$ Q $(-\hat{i}+\hat{j}+\hat{k}) = \vec{q}$

P $(\hat{i}+2\hat{i}-\hat{k}) = \vec{p}$

$$\vec{r} = \frac{2\vec{q}+\vec{p}}{2+1}$$

internal Division

$\vec{p} = \hat{i}+2\hat{j}-\hat{k}$

$\vec{q} = -\hat{i}+\hat{j}+\hat{k}$

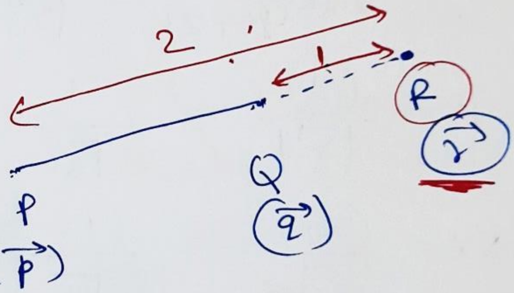
$$\vec{r} = \frac{2\vec{q}+\vec{p}}{2+1}$$

$$\vec{r} = \frac{2(-\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$\vec{r} = \frac{(-2+1)\hat{i}+(2+2)\hat{j}+(2-1)\hat{k}}{3}$$

$$\boxed{\vec{r} = \frac{-\hat{i}}{3}+\frac{4}{3}\hat{j}+\frac{\hat{k}}{3}}$$ ✓

(ii) external Division.



$$\vec{r} = \frac{2\vec{q}-\vec{p}}{2-1}$$

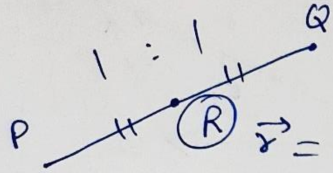
$$= \frac{2(-\hat{i}+\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-\hat{k})}{\textcircled{1}}$$

$$= (-2-1)\hat{i}+\overset{0}{\cancel{(2-2)}}\hat{j}+(2+1)\hat{k}$$

$$= \underline{-3\hat{i}+3\hat{k}}$$ ✓

$$\underline{(-3, 0, 3)}$$

Q.16 Find the position Vector of the mid point of the vector joining the points P(2,3,4) & Q(4,1,-2)



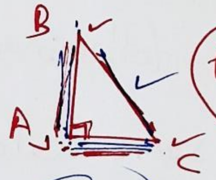
$\vec{q} = 4\hat{i}+\hat{j}-2\hat{k}$

$\vec{r} = \vec{OR} = \vec{r} = \frac{\vec{p}+\vec{q}}{2} = \frac{(2\hat{i}+3\hat{j}+4\hat{k}) + (4\hat{i}+\hat{j}-2\hat{k})}{2}$

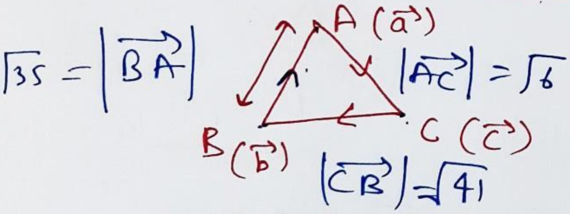
$\vec{p} = 2\hat{i}+3\hat{j}+4\hat{k}$

$= 3\hat{i}+2\hat{j}+\hat{k}$

Q.17 Show that the ~~Vectors~~ points A, B & C with position vectors, $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$, respectively form the vertices of a right angled triangle.



Pythagoras Theorem



$\vec{BA} =$ Final - initial

$\vec{a} - \vec{b}$

$|\vec{BA}| = |\vec{a} - \vec{b}| = |3\hat{i}-4\hat{j}-4\hat{k}-2\hat{i}+\hat{j}-\hat{k}|$

$= |\hat{i}-3\hat{j}-5\hat{k}| = \sqrt{1^2+9+25} = \sqrt{35}$

$|\vec{AC}| = |\vec{c}-\vec{a}| = |\hat{i}-3\hat{j}-5\hat{k}-3\hat{i}+4\hat{j}+4\hat{k}| = |-2\hat{i}+\hat{j}-\hat{k}|$

$= \sqrt{4+1+1} = \sqrt{6}$

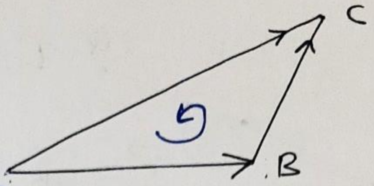
$|\vec{CB}| = |\vec{b}-\vec{c}| = |2\hat{i}-\hat{j}+\hat{k}-\hat{i}+3\hat{j}+5\hat{k}| = |\hat{i}+2\hat{j}+6\hat{k}|$

CB

$= \sqrt{1+4+36} = \sqrt{41}$

$41 = 35+6 \Rightarrow (\sqrt{41})^2 = (\sqrt{35})^2 + (\sqrt{6})^2 \Rightarrow \boxed{CB^2 = BA^2 + AC^2}$

Q.18 In triangle ABC (given fig.), which of the following is not true:

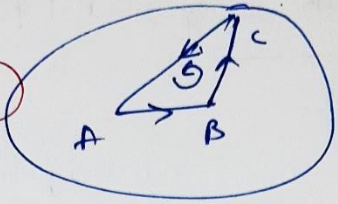


(A) $\vec{AB}+\vec{BC}+\vec{CA}=\vec{0}$ T

(B) $\vec{AB}+\vec{BC}-\vec{AC}=\vec{0}$ T

(C) $\vec{AB}+\vec{BC}-\vec{CA}=\vec{0}$ F

(D) $\vec{AB}-\vec{CB}+\vec{CA}=\vec{0}$ T



$\vec{AB}+\vec{BC}+\vec{CA}=\vec{0}$

Q.19 If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

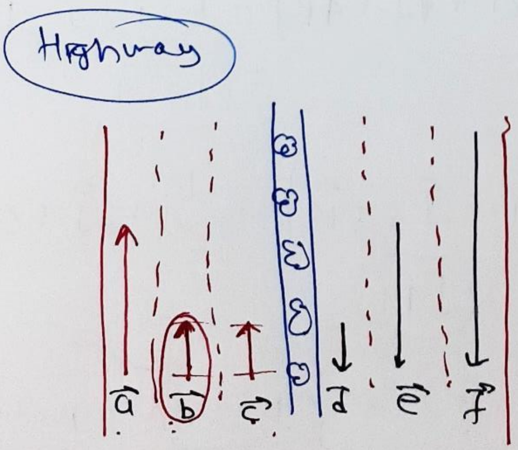
T (A) $\vec{b}=\lambda\vec{a}$, for some scalar λ.

F (B) $\vec{a}=\pm\vec{b}$ $|\vec{a}|=|\pm\vec{b}|$ $|\vec{a}|=|\vec{b}|$

F (C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional.

F (D) both the vectors $\vec{a}$ & $\vec{b}$ have same direction, but different magnitudes.

Ans: B, C, D



$\vec{b}$

$\vec{a},\vec{c},\vec{d},\vec{e},\vec{f}$

⇓

$\lambda\vec{b}$

All vectors

⇓

Collinear

Parallel

[redacted] | Class 12 maths | Vector Algebra

Product of Two Vectors

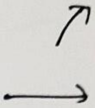
Today

Scalar (Dot) Product

$\vec{a}\cdot\vec{b} = \boxed{\text{Scalar}}$

Vector (Cross) Product

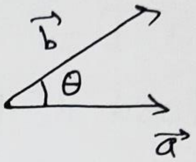
$\vec{a}\times\vec{b} = \boxed{\text{Vector}}$



Scalar (or Dot) Product : (अदिश गुणनफल)

$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$$

Scalar

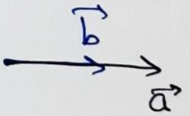


Note. ① Either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, $\theta=$ not defined,

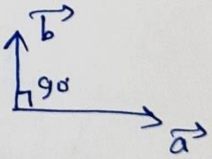
$\boxed{\vec{a}\cdot\vec{b}=0}$

② Let $\vec{a}$ & $\vec{b}$ be non zero vectors.

(I) $\theta=0°$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 0° = |\vec{a}||\vec{b}|$

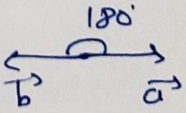


(II) $\theta=90°$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\underbrace{\cos 90°}_{0} = 0$



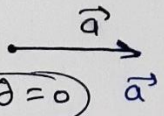
$\boxed{\vec{a}\cdot\vec{b}=0} \Leftrightarrow \boxed{\vec{a}\perp\vec{b}}$

(III) $\theta=180°$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\overset{-1}{\cos 180°} = -|\vec{a}||\vec{b}|$



~~$\vec{a}\cdot\vec{b}$~~

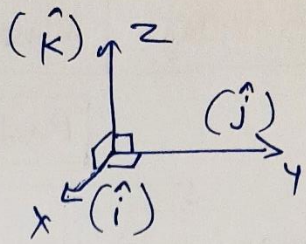
$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos 0° = |\vec{a}|^2$



$\theta=0$

$\boxed{\vec{a}\cdot\vec{a} = |\vec{a}|^2}$ ☆

③



$\hat{i}\cdot\hat{i} = |\hat{i}|^2 = 1^2 = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

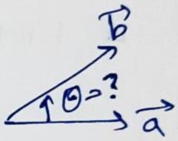
$$\boxed{\hat{i}\cdot\hat{j} = 0 = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i}}$$

④ Commutative (क्रम विनिमेय)

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$|\vec{a}||\vec{b}|\cos\theta \quad |\vec{b}||\vec{a}|\cos\theta$

⑤ Dot Product ⟶ Angle



$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\underset{\uparrow}{\theta}$

$$\boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}} \qquad \boxed{\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)}$$

⑥ $\vec{a}\cdot(\vec{b}\pm\vec{c}) = \vec{a}\cdot\vec{b} \pm \vec{a}\cdot\vec{c}$

$\lambda(\vec{a}\cdot\vec{b}) = (\lambda\vec{a})\cdot\vec{b} = \vec{a}\cdot(\lambda\vec{b})$

Note: $(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$= a_1b_1 + a_2b_2 + a_3b_3$

e.g. Find angle 'θ' between $\vec{a} = \hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+\hat{k}$.

↓ Dot Product

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\theta$$

$$\Rightarrow (\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k}) = \sqrt{1^2+1^2+(-1)^2}\cdot\sqrt{1+1+1}\cdot\cos\theta$$

$$\Rightarrow |\hat{i}|^2 - |\hat{j}|^2 - |\hat{k}|^2 = \sqrt{3}\cdot\sqrt{3}\cdot\cos\theta$$

$$\Rightarrow \cancel{1} - \cancel{1} - 1 = 3\cdot\cos\theta$$

$$\Rightarrow \boxed{\cos\theta = -1/3}$$ ✓

e.g. If $\vec{a} = 5\hat{i}-\hat{j}-3\hat{k}$ & $\vec{b} = \hat{i}+3\hat{j}-5\hat{k}$, show that $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$.

$$\vec{a}+\vec{b} = 6\hat{i}+2\hat{j}-8\hat{k}$$

$$\vec{a}-\vec{b} = 4\hat{i}-4\hat{j}+2\hat{k}$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (6\hat{i}+2\hat{j}-8\hat{k})\cdot(4\hat{i}-4\hat{j}+2\hat{k})$$

$$= 24 - 8 - 16 = 24 - 24 = 0$$

$$\boxed{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0} \Rightarrow (\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$$

☆

e.g. Find $|\vec{a}-\vec{b}|$, if two vectors $\vec{a}$ & $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$. $|\vec{a}-\vec{b}|$

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$= \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

$$= |\vec{a}|^2 - 2(\vec{a}\cdot\vec{b}) + |\vec{b}|^2$$

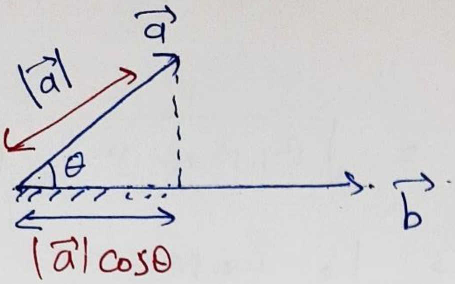
$$= 4 - 2(4) + 9 = 5$$

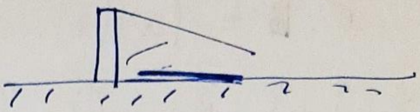
$$\begin{array}{r} 13 \\ -\ 8 \\ \hline 5 \end{array}$$

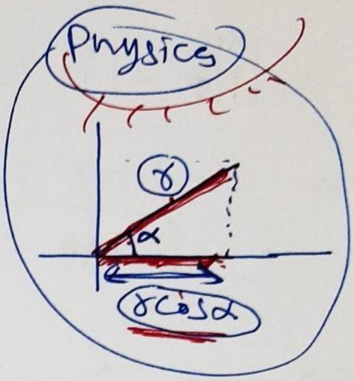
$| \; | = ⊕$

$$|\vec{a}-\vec{b}|^2 = 5 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{5}$$

Application of Dot Product = Projection of a Vector on Another Vector







$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

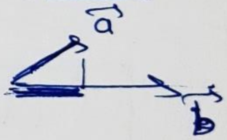
$$\Rightarrow \boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$$

length of projection of $\vec{a}$ on $\vec{b}$

$$= |\vec{a}|\cos\theta$$

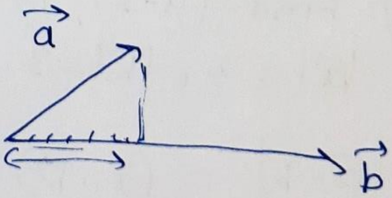
$$= \cancel{|\vec{a}|}\cdot\left(\frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}|\vec{b}|}\right) = \frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|} = \vec{a}\cdot\hat{b}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|}$$



e.g. Find the projection of the vector $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ on vector $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$.

Projection of $\vec{a}$ on $\vec{b}$



$$= \vec{a}\cdot\hat{b} = \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}$$

$$= \frac{(2\hat{i}+3\hat{j}+2\hat{k})\cdot(\hat{i}+2\hat{j}+\hat{k})}{\sqrt{1^2+2^2+1^2}} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

e.g. Three vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ satisfy the condition

☆ $\boxed{\vec{a}+\vec{b}+\vec{c}=\vec{0}}$. Evaluate the quantity

☆ $\underbrace{\mu = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}}_{=\,?}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ & $|\vec{c}|=2$.

Ans.

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = |\vec{0}| = 0$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 0^2 = 0$$

$$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\Rightarrow \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}\cdot\vec{c} = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}) + 2\,\vec{a}\cdot\vec{c} + 2\,\vec{b}\cdot\vec{c} = 0$$

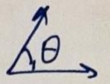
$$\Rightarrow 3^2 + 4^2 + 2^2 + 2\big(\underbrace{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}}_{\mu}\big) = 0$$

$$\Rightarrow 9 + 16 + 4 + 2(\mu) = 0$$

$$\Rightarrow 2\mu = -29 \Rightarrow \boxed{\mu = -\frac{29}{2}}$$

Exercise-10.3

Scalar Product / (Dot Product)



$= \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Q.1 Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$.

Ans. $\vec{a}\cdot\vec{b}=\sqrt{6}$ (given)

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta=\sqrt{6}$

$\Rightarrow \underbrace{\sqrt{3}\cdot 2\cos\theta}=\sqrt{6}=\sqrt{2}\cdot\sqrt{3}$

$\Rightarrow \cos\theta=\frac{\cancel{\sqrt{2}}\cdot\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\cdot\cancel{2}\sqrt{2}}$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$ $\boxed{\theta\in[0,\pi]}$

$\boxed{\theta=\frac{\pi}{4}}$

Q.2 Find the angle between the vectors $\underset{\vec{a}}{\hat{i}-2\hat{j}+3\hat{k}}$ and $3\hat{i}-2\hat{j}+\hat{k}$. $\rightarrow \vec{b}$

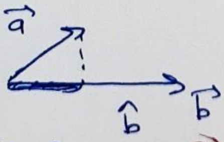
$\vec{a}\cdot\vec{b}=|\vec{a}|.|\vec{b}|\cos\theta$

$\Rightarrow (\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})=\sqrt{1+4+9}.\sqrt{9+4+1}.\cos\theta$

$\Rightarrow (3+4+3)=\sqrt{14}.\sqrt{14}\cos\theta \Rightarrow 10=14\cos\theta$

$\Rightarrow \cos\theta=\frac{10}{14}=\frac{5}{7} \Rightarrow \boxed{\theta=\cos^{-1}\left(\frac{5}{7}\right)}$

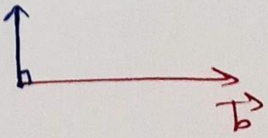
Q.3 Find the projection of the vector $\underset{\vec{a}}{\hat{i}-\hat{j}}$ on the vector $\hat{i}+\hat{j}$. $\rightarrow \vec{b}$



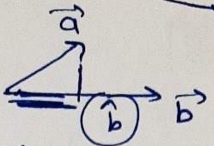
$\hat{b}=\frac{\vec{b}}{|\vec{b}|}$

Projection of $\vec{a}$ on $\vec{b}$ $=\vec{a}.\hat{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$=\frac{(\hat{i}-\hat{j}).(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}=\frac{1-1}{\sqrt{2}}=\frac{0}{\sqrt{2}}=0$



Q.4 Find the projection of the vector $\underbrace{\hat{i}+3\hat{j}+7\hat{k}}_{\vec{a}}$ on the vector $7\hat{i}-\hat{j}+8\hat{k} \rightarrow \vec{b}$



Projection of $\vec{a}$ on $\vec{b}$ $= \vec{a}\cdot\hat{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(\hat{i}+3\hat{j}+7\hat{k})\cdot(7\hat{i}-\hat{j}+8\hat{k})}{\sqrt{49+1+64}} = \frac{7-3+56}{\sqrt{114}} = \frac{60}{\sqrt{114}} \checkmark$$

Q.5 Show that each of the given three vectors is a unit vector (length): $\underbrace{\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})}_{\vec{a}}$, $\underbrace{\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})}_{\vec{b}}$, $\underbrace{\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})}_{\vec{c}}$.

Also, show that they are mutually perpendicular to each other.

$$|\vec{a}| = \left|\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}\right| = \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} = \sqrt{\frac{49}{49}} = 1 \text{ unit}$$

$$|\vec{b}| = \left|\frac{3\hat{i}}{7}-\frac{6\hat{j}}{7}+\frac{2}{7}\hat{k}\right| = \sqrt{\frac{9}{49}+\frac{36}{49}+\frac{4}{49}} = \sqrt{\frac{49}{49}} = 1 \text{ unit}$$ —

$|\vec{c}| = 1$ unit —

mutually Perpendicular to each other

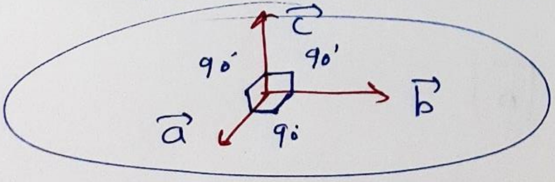
$\vec{a}\cdot\vec{b}=0$ $\vec{b}\cdot\vec{c}=0$ $(\vec{c}\cdot\vec{a})=0$

$$\vec{a}\cdot\vec{b} = \frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})\cdot\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}) = \frac{1}{49}(6-18+12)=0$$

$\vec{b} \perp \vec{a}$

$$\vec{b}\cdot\vec{c} = \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})\cdot\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k}) = \frac{1}{49}(18-12-6)=0$$

$$\vec{c}\cdot\vec{a} = \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})\cdot\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}) = \frac{1}{49}(12+6-18)=0$$



Q.6 Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$. (+)

$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$

Ans. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} - \cancel{\vec{a}\cdot\vec{b}} + \cancel{\vec{b}\cdot\vec{a}} - \vec{b}\cdot\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$ — ①

$|\vec{a}| = 8|\vec{b}|$ — ②

By eqⁿ ① & ②.

$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}|^2 = \frac{8}{63} = \frac{4\times2}{9\times7}$

$\Rightarrow \boxed{|\vec{b}| = \sqrt{\frac{4\times2}{9\times7}} = \frac{2\sqrt{2}}{3\sqrt{7}}}$

$|\vec{a}| = 8|\vec{b}| = 8\times\left(\frac{2\sqrt{2}}{3\sqrt{7}}\right)$

$\boxed{|\vec{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}}$

Q.7 Evaluate the product $(3\vec{a} - 5\vec{b})\cdot(2\vec{a}+7\vec{b})$.

$\Rightarrow 6\,\vec{a}\cdot\vec{a} + 21(\vec{a}\cdot\vec{b}) - 10(\vec{b}\cdot\vec{a}) - 35\,\vec{b}\cdot\vec{b}$ $\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$

$= 6|\vec{a}|^2 + 11\,\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$ ✓

Q.8 Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

$\boxed{|\vec{a}| = |\vec{b}|}$ $\theta = 60^\circ$ $\cos 60^\circ = \cos\theta = \frac{1}{2}$ $\vec{a}\cdot\vec{b} = \frac{1}{2}$

$\vec{a}\cdot\vec{b} = \frac{1}{2}$ $|\vec{a}| = |\vec{b}| > 0$

$\Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta = \frac{1}{2}$

$\Rightarrow |\vec{a}|\cdot|\vec{a}|\left(\cancel{\frac{1}{2}}\right) = \cancel{\frac{1}{2}}$

$\Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1 = |\vec{b}|$

Q.9 Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 12$.

$|\vec{a}| = 1$

$\vec{a}\cdot\vec{a} = |\vec{a}|^2$

$\vec{x}\cdot\vec{a} = \vec{a}\cdot\vec{x}$

$\Rightarrow \vec{x}\cdot\vec{x} + \cancel{\vec{x}\cdot\vec{a}} - \cancel{\vec{a}\cdot\vec{x}} - \vec{a}\cdot\vec{a} = 12$

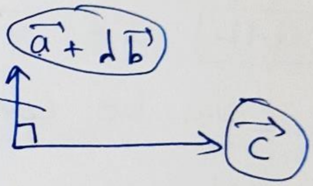
$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1^2 = 12$

$\Rightarrow |\vec{x}|^2 = 13 \Rightarrow \boxed{|\vec{x}| = \sqrt{13}}$

$|\vec{x}| \geq 0$

Q.10 If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $(\vec{a}+\lambda\vec{b})$ is perpendicular to $\vec{c}$, then find the value of λ.



$(\vec{a}+\lambda\vec{b})\cdot\vec{c} = 0$

$\Rightarrow [2\hat{i}+2\hat{j}+3\hat{k} + \lambda(-\hat{i}+2\hat{j}+\hat{k})]\cdot(3\hat{i}+\hat{j}) = 0$

$\Rightarrow [\hat{i}(2-\lambda) + \hat{j}(2+2\lambda) + \hat{k}(3+\lambda)]\cdot(3\hat{i}+\hat{j}) = 0$

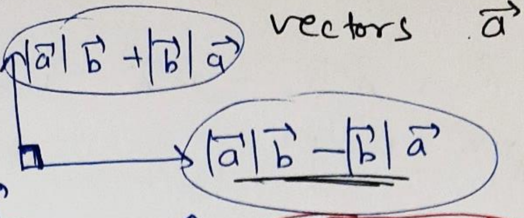
$\Rightarrow (2-\lambda).3 + (2+2\lambda).1 + 0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow 8 - \lambda = 0$

$\Rightarrow \boxed{\lambda = 8}$

$\boxed{Q.11}$ Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ & $\vec{b}$.



$\vec{a}.\vec{b} = \vec{b}.\vec{a}$

$(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}).(|\vec{a}|\vec{b} - |\vec{b}|\vec{a})$

$= |\vec{a}|^2\,\vec{b}.\vec{b} - \cancel{|\vec{a}||\vec{b}|\,\vec{b}.\vec{a}}$

$+ \cancel{|\vec{b}||\vec{a}|\,\vec{a}.\vec{b}} - |\vec{b}|^2\,\vec{a}.\vec{a}$

$\Rightarrow \cancel{|\vec{a}|^2|\vec{b}|^2} - \cancel{|\vec{b}|^2|\vec{a}|^2}$

$= 0$

$\boxed{Q.12}$ If $\vec{a}.\vec{a} = 0$ and $\vec{a}.\vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

magnitude → length of vector

$\vec{a}.\vec{a} = 0$

$\Rightarrow |\vec{a}|^2 = 0$

$\Rightarrow |\vec{a}| = 0$

length of $\vec{a} = 0$

$\vec{a}$ = Zero vector

$\vec{a}.\vec{b} = 0$ (Given)

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = 0$

$\downarrow$ 0

$\Rightarrow (0).|\vec{b}|\cos\theta = 0$

$0 \times (\ ) = 0$

$\vec{b} \rightarrow$ any vector

Exercise 10.3

Scalar Product (Dot Product) θ

$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$$

Q.13 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\boxed{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}}$.

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\boxed{|\vec{x}|^2 = \vec{x}\cdot\vec{x}}$$

Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ $\quad \boxed{|\vec{0}|=0}$

$$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c}}$$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow \boxed{\vec{a}\cdot\vec{a}} + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \boxed{\vec{b}\cdot\vec{b}} + \vec{b}\cdot\vec{c}$
$\quad + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \boxed{\vec{c}\cdot\vec{c}} = 0$

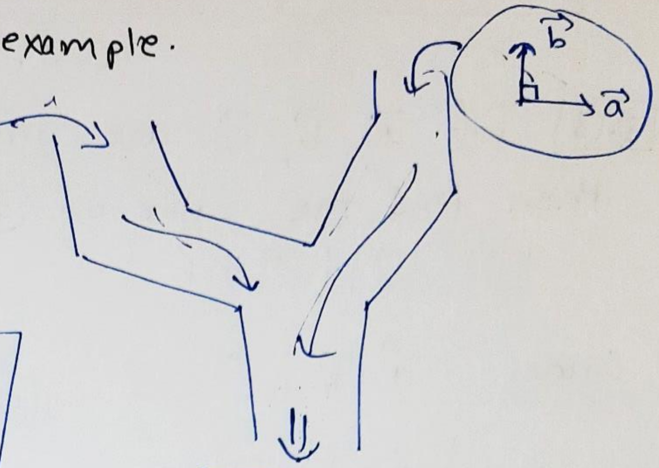
$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + \underbrace{2\,\vec{a}\cdot\vec{b} + 2\,\vec{c}\cdot\vec{a} + 2\,\vec{b}\cdot\vec{c}} = 0$

$\Rightarrow \underbrace{1+1+1} + 2(\underbrace{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}}) = 0$

$$\Rightarrow \boxed{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-3}{2}}$$

Q.14 If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\cdot\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$



$\boxed{\vec{a}\cdot\vec{b}=0}$

Statement Given

If either vector $\vec{a}=0$ or $\vec{b}=0$ then $\vec{a}\cdot\vec{b}=0$

Converse ~~Statement~~ Statement:→

If $\vec{a}\cdot\vec{b}=0$, then either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$

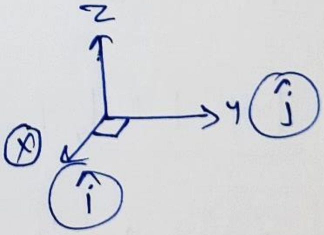
↓

False

Let $\vec{a}=\hat{i}$

$\vec{b}=\hat{j}$

$\vec{a}\cdot\vec{b}=\hat{i}\cdot\hat{j}=0$



$\vec{a}=\hat{i}\neq\vec{0}$

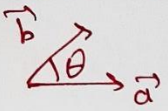
$\vec{b}=\hat{j}\neq\vec{0}$

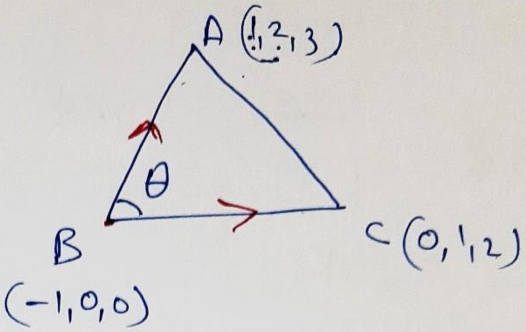
Q.15 If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0), (0,1,2), respectively, then find ∠ABC.

A B C

Dot Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$





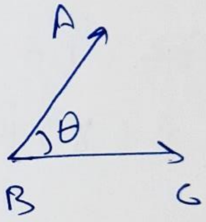
Final - initial

$\vec{BA} = (1+1)\hat{i} + (2-0)\hat{j} + (3-0)\hat{k}$

$= 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{BC} = (0+1)\hat{i} + (1-0)\hat{j} + (2-0)\hat{k}$

$= \hat{i} + \hat{j} + 2\hat{k}$



Dot Product

$\vec{BC}\cdot\vec{BA} = |\vec{BC}|\,|\vec{BA}|\cos\theta$

$\Rightarrow \underline{(\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}+2\hat{j}+3\hat{k})} = \sqrt{1^2+1^2+2^2}\cdot\sqrt{4+4+9}\cdot\cos\theta$

$\Rightarrow (2+2+6) = \sqrt{6}\,\sqrt{17}\cos\theta$

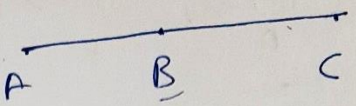
$\Rightarrow 10 = \sqrt{102}\cdot\cos\theta$

$\Rightarrow \frac{10}{\sqrt{102}} = \cos\theta \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)}$

$\boxed{Q.16}$ Show that the points A(1,2,7), B(2,6,3) and C(3,10,−1) are collinear.

Vector

Final − Initial



$$\underset{|\vec{AB}|}{\underset{\uparrow}{AB}} + \underset{|\vec{BC}|}{\underset{\uparrow}{BC}} = \underset{|\vec{AC}|}{\underset{\uparrow}{AC}}$$

$$\vec{AB} = (1)\hat{i} + (4)\hat{j} + (-4)\hat{k}$$

$$\Rightarrow \vec{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$|\vec{AB}| = \sqrt{1+16+16} = \sqrt{33}$$

$$\vec{BC} = (1)\hat{i} + (4)\hat{j} + (-4)\hat{k} \Rightarrow \vec{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$|\vec{BC}| = \sqrt{1+16+16} = \sqrt{33}.$$

$$\vec{CA} = (-2)\hat{i} + (-8)\hat{j} + (8)\hat{k} \Rightarrow \vec{CA} = -2\hat{i} - 8\hat{j} + 8\hat{k}$$

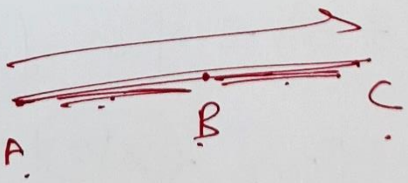
$$|\vec{CA}| = \sqrt{4+64+64}$$

$$= \sqrt{4(1+16+16)}$$

$$= 2\sqrt{33}$$

$$2\sqrt{33} = \sqrt{33} + \sqrt{33}$$

$$|\vec{CA}| = |\vec{BC}| + |\vec{AB}|$$

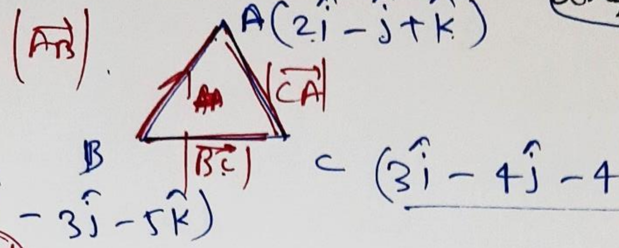


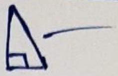
Collinear

points.

A, B, C

Q.17 Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.





$\updownarrow$

P.G.T

Pythagoras

Final - initial

$\vec{AB} = (\hat{i}-3\hat{j}-5\hat{k}) - (2\hat{i}-\hat{j}+\hat{k})$

$= -\hat{i}-2\hat{j}-6\hat{k}$

$\vec{BC} = (3\hat{i}-4\hat{j}-4\hat{k}) - (\hat{i}-3\hat{j}-5\hat{k})$

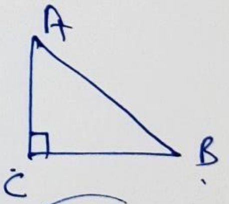
$= 2\hat{i}-\hat{j}+\hat{k}$

$\vec{CA} = (2\hat{i}-\hat{j}+\hat{k}) - (3\hat{i}-4\hat{j}-4\hat{k}) = -\hat{i}+3\hat{j}+5\hat{k}$

$|\vec{AB}| = \sqrt{(-1)^2+(-2)^2+(-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$|\vec{BC}| = \sqrt{4+1+1} = \sqrt{6}$

$|\vec{CA}| = \sqrt{1+9+25} = \sqrt{35}$



Hypo.

P.G.T.

$41 = 6 + 35$

$\Rightarrow (\sqrt{41})^2 = (\sqrt{6})^2 + (\sqrt{35})^2 \Rightarrow \boxed{|\vec{AB}|^2 = |\vec{BC}|^2 + |\vec{CA}|^2}$

Q.18 If $\vec{a}$ is a non zero vector of magnitude 'a' and λ a non zero scalar, then $\lambda\vec{a}$ is unit vector if

(A) $\lambda = 1$ (B) $\lambda = -1$

(C) $a = |\lambda|$ [✓ (D)] $a = 1/|\lambda|$

$|\vec{a}| = a$

$\lambda \rightarrow$ scalar $\in R - \{0\}$

$\lambda\vec{a} \rightarrow$ unit vector

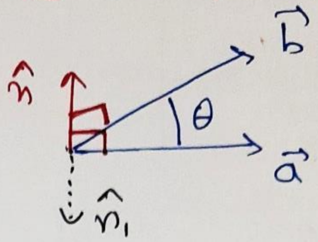
$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda|\,|\vec{a}| = 1$

$\Rightarrow |\lambda| . a = 1$

$a = \frac{1}{|\lambda|}$

Vector (or Cross) Product (सदिश गुणनफल)



$$\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}}$$

$\vec{a}, \vec{b}, \hat{n}$ → Follow → Right Hand Thumb Rule (RHTR)

(इसी order में)

$\hat{n}$ = unit vector perpendicular to the vectors $\vec{a}$ & $\vec{b}$ obtained by RHTR

$$\vec{b}\times\vec{a} = |\vec{b}||\vec{a}|\sin\theta\,\hat{n}_1$$

$\vec{b}, \vec{a}, \hat{n}_1$ → follow → Right Hand Thumb Rule.

Here $\hat{n}$ & $\hat{n}_1$ have same magnitude (1) but opposite direction.

$$\hat{n}_1 = -\hat{n}$$

← →

$$\vec{b}\times\vec{a} = -|\vec{b}||\vec{a}|\sin\theta\,\hat{n}$$

$$\boxed{\vec{b}\times\vec{a} = -(\vec{a}\times\vec{b})}$$

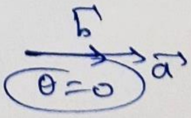
$$\vec{a}\times\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{b}\times\vec{a} = -\hat{i} + 2\hat{j} - \hat{k}$$

Cases:

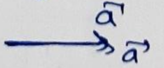
$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$\boxed{\theta = 0}$

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\underset{0}{\sin\theta}\,\hat{n} = 0$$

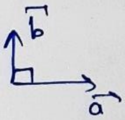


$$\boxed{\vec{a}\times\vec{a} = 0}$$



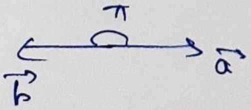
$\theta = 0$

$\theta = 90°$



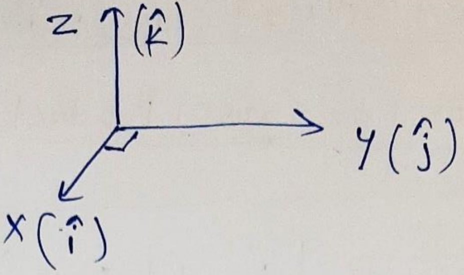
$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\hat{n}$$

$\theta = 180°$



$$\vec{a}\times\vec{b} = 0$$

$\boxed{\vec{a}\times\vec{a}=0}$

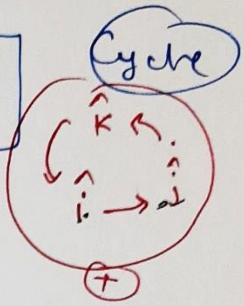


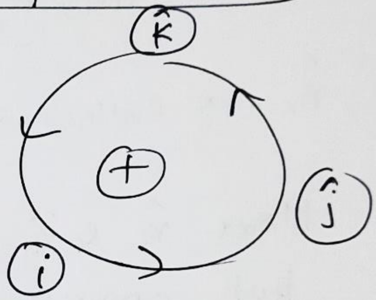
$\hat{i}\times\hat{i}=0$

$\hat{j}\times\hat{j}=0$

$\hat{k}\times\hat{k}=0$

$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{i}=\hat{j}$
$\hat{j}\times\hat{i}=-\hat{k}$	$\hat{k}\times\hat{j}=-\hat{i}$	$\hat{i}\times\hat{k}=-\hat{j}$





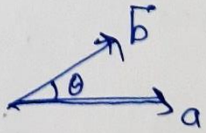
Note: $\vec{a}\times\vec{b}=|\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$ Cross product.

magnitude.

$$|\vec{a}\times\vec{b}| = \Big|\underbrace{|\vec{a}||\vec{b}|\sin\theta}_{\text{Scalar}}\ \underset{\uparrow \text{ vector Unit}}{\hat{n}}\Big|$$

$$\Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta\ \underbrace{|\hat{n}|}_{\to 1}$$

$$\Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$



$$\Rightarrow \boxed{\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}}$$

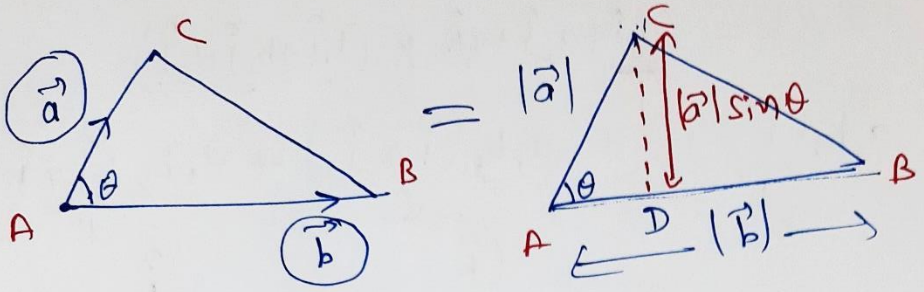
Angle

∠

Dot

Geometric application of Vector(or Cross Product) :→ $\boxed{\vec{a} \times \vec{b}}$

Area of Δ, area of Parallelogram ▱



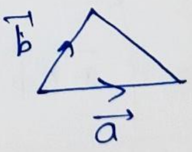
$$\text{Area of } \Delta ABC = \frac{1}{2} \text{ Base} \times \text{Height}$$

$$= \frac{1}{2} (AB) \times CD$$

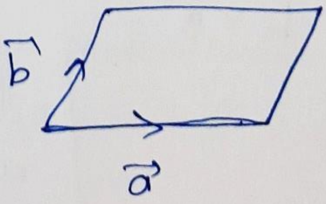
$$= \frac{1}{2} \boxed{|\vec{b}| \times |\vec{a}| \sin\theta}$$

$$= \frac{1}{2} \left| \underset{\uparrow}{\vec{a} \times \vec{b}} \right| .$$

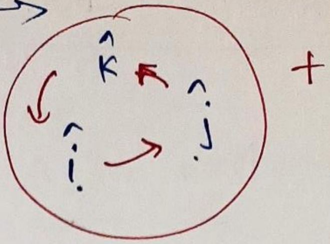
- Area of Triangle whose 2 sides are given by $\vec{a}$ & $\vec{b}$ $= \frac{1}{2} |\vec{a} \times \vec{b}|$



- Area of a Parallelogram whose 2 adjacent sides are given by $\vec{a}$ & $\vec{b}$ $= |\vec{a} \times \vec{b}|$



$\vec{a} \times \vec{b}$ को Solve करने के 2 तरीके $\longrightarrow$



$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

(i) method.

$$\vec{a} \times \vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$

$$= \underbrace{a_1b_1(\hat{i}\times\hat{i})}_{0} + \underbrace{a_1b_2(\hat{i}\times\hat{j})}_{\hat{k}} + \underbrace{a_1b_3(\hat{i}\times\hat{k})}_{-\hat{j}}$$

$$+ \underline{a_2b_1(-\hat{k})} + \underline{0} + \underline{a_2b_3(+\hat{i})}$$

$$+ \underline{a_3b_1(+\hat{j})} + \underline{a_3b_2(-\hat{i})} + \underline{0}$$

$$\boxed{\vec{a}\times\vec{b} = (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}}$$

II - method

$$\vec{a} \times \vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$

$$\Rightarrow \boxed{\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}} \rightarrow \text{expand}$$

Determinant

3×3

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

e.g. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ &
$\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$.

Ans. $\vec{a}\times\vec{b}=(2\hat{i}+\hat{j}+3\hat{k})\times(3\hat{i}+5\hat{j}-2\hat{k})$

$=\underline{10\hat{k}+4}\hat{j}-\underline{3\hat{k}-2}\hat{i}+\underline{9\hat{j}+15}\hat{i}$

$\vec{a}\times\vec{b}=-17\hat{i}+13\hat{j}+7\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{(-17)^2+13^2+7^2}$

$=\sqrt{289+169+49}=\sqrt{507}$

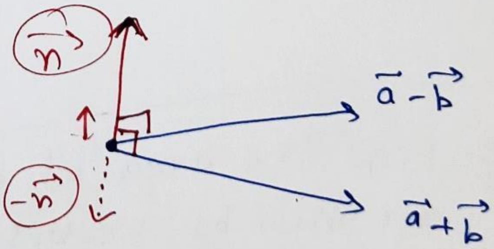
e.g. Find a unit vector perpendicular to each of ☆ the $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$,
$\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

Ans. $\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

Cross Product
‖
Vector Product

Right Hand Thumb Rule

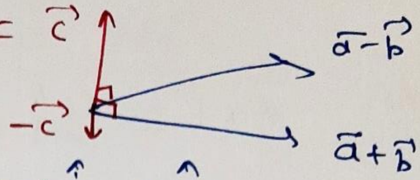


$\underline{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}$ = vector which is perpendicular to the both $(\vec{a}+\vec{b})$ & $(\vec{a}-\vec{b})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=\begin{matrix}\hat{i}(-6+4)\\ -\hat{j}(-4-0)\\ +\hat{k}(-2-0)\end{matrix}=\begin{matrix}-2\hat{i}\\ +4\hat{j}\\ -2\hat{k}\end{matrix}$$

$\vec{c} = \underline{(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})} = -2\hat{i}+4\hat{j}-2\hat{k}$

$\underline{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \vec{c}}$



Unit vector $\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{-2\hat{i}+4\hat{j}-2\hat{k}}{\sqrt{4+16+4}}$

$$= \frac{-2\hat{i}+4\hat{j}-2\hat{k}}{\sqrt{24}}$$

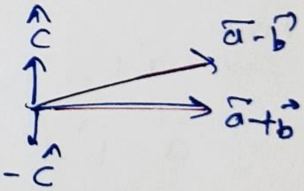
$$\hat{c} = \frac{-2\hat{i}+4\hat{j}-2\hat{k}}{2\sqrt{6}} = \boxed{\frac{-\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}}$$ ✓

One more Possibility. $= -\hat{c} = \frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}$ ✓

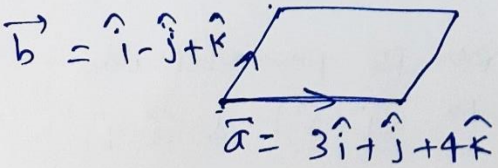
Unit vector

Perpendicular to $(\vec{a}+\vec{b})$ & $(\vec{a}-\vec{b})$

$$= \pm\left(\frac{-\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}\right)$$



e.g. Find the area of a parallelogram whose sides are given by vectors $\vec{a} = 3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+\hat{k}$



Required Area $= |\vec{a}\times\vec{b}|$

$$= |5\hat{i}+\hat{j}-4\hat{k}|$$

$$= \sqrt{25+1+16}$$

$$= \sqrt{42} \text{ Sq. units.}$$

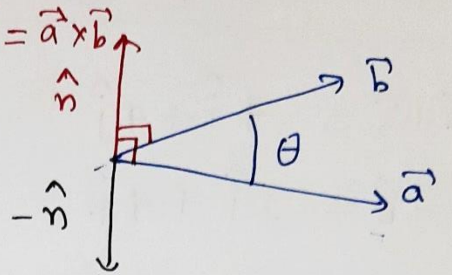
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(5) - \hat{j}(-1) + (-4)\hat{k}$$

Exercise 10.4 Vector (or Cross) Product.

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta.\hat{n}$

$\left(\vec{a}, \vec{b}, \hat{n} \rightarrow \text{Right Hand thumb rule}\right)$

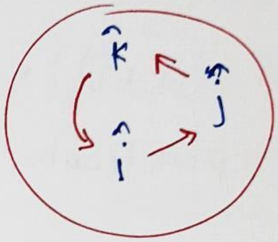


$\boxed{\vec{a}\times\vec{a} = 0}$

$(\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k})$

$\vec{b}\times\vec{a} = -\vec{a}\times\vec{b} = -\hat{n}$

$$\begin{pmatrix}\hat{i}\times\hat{j} = \hat{k}\\ \hat{j}\times\hat{k} = \hat{i}\\ \hat{k}\times\hat{i} = \hat{j}\end{pmatrix} \quad \begin{pmatrix}\hat{j}\times\hat{i} = -\hat{k}\\ \hat{k}\times\hat{j} = -\hat{i}\\ \hat{i}\times\hat{k} = .\hat{j}\end{pmatrix}$$



Exercise 10.4 Class 12

Q.1 Find $|\vec{a}\times\vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ anb $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -7 & 7\\ 3 & -2 & 2\end{vmatrix} = \hat{i}\begin{vmatrix}-7 & 7\\ -2 & 2\end{vmatrix} - \hat{j}\begin{vmatrix}1 & 7\\ 3 & 2\end{vmatrix} + \hat{k}\begin{vmatrix}1 & -7\\ 3 & -2\end{vmatrix}$$

$$\vec{a}\times\vec{b} = 0.\hat{i} - \hat{j}(-19) + \hat{k}(19) = 0.\hat{i} + 19\hat{j} + 19\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{0 + 19^2 + 19^2}$$

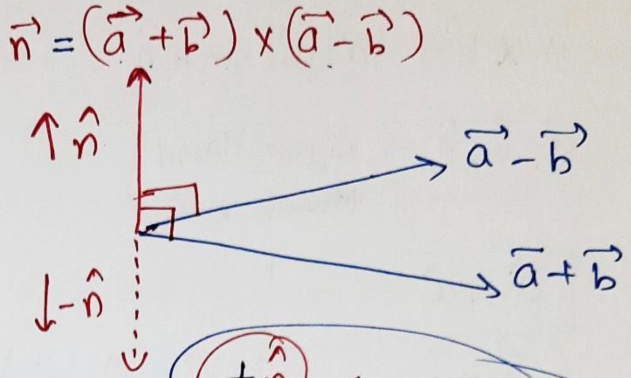
$$= \sqrt{2\times19^2} = 19\sqrt{2}$$

$\hat{n}$ ② ① $\vec{n}$

Q.2 Find a unit vector perpendicular to each of the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

$$\vec{a}+\vec{b} = 4\hat{i} + 4\hat{j}$$

$$\vec{a}-\vec{b} = 2\hat{i} + 4\hat{k}$$



$\pm\hat{n}$ ✓ = Answers

$\vec{n} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})$ will be perpendicular to both $(\vec{a}+\vec{b})$ & $(\vec{a}-\vec{b})$.

$$\vec{n} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = (4\hat{i} + 4\hat{j} + 0\hat{k}) \times (2\hat{i} + 0.\hat{j} + 4\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \begin{array}{l} \hat{i}(16) \\ -\hat{j}(16) \\ +\hat{k}(-8) \end{array}$$

$$\vec{n} = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{256 + 256 + 64}} = \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$$

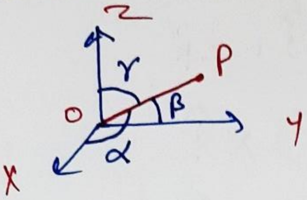
$$\boxed{\hat{n} = \frac{2\hat{i}}{3} - \frac{2\hat{j}}{3} - \frac{\hat{k}}{3}}$$

Perpendicular unit vectors $= \pm\hat{n} = \pm\left(\frac{2\hat{i}}{3} - \frac{2\hat{j}}{3} - \frac{\hat{k}}{3}\right)$

$$= \pm\frac{2\hat{i}}{3} \mp \frac{2\hat{j}}{3} \mp \frac{\hat{k}}{3}$$ ✓

$\boxed{Q.3}$ If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$ (x), $\frac{\pi}{4}$ with $\hat{j}$ (y) and ~~and~~ an acute angle θ with $\hat{k}$ (z), then find θ and hence, the components of $\vec{a}$.

I-method,



$\vec{OP} = \vec{a}$

$\alpha, \beta, r \rightarrow$ Direction angles.

$\cos\alpha, \cos\beta, \cos r \rightarrow$ Direction Cosines.

$|\vec{a}| = 1$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2 r = 1}$$

Unit vector $\boxed{\vec{a} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos r\,\hat{k}}$

ATQ. $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$, $r = \theta$

$\because \cos^2\alpha + \cos^2\beta + \cos^2 r = 1$

$\Rightarrow \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2}$

$\theta \rightarrow$ acute

$\cos\theta = \frac{1}{2} = \cos r$

$\boxed{\theta = 60^\circ = \frac{\pi}{3}}$

$\theta = \frac{\pi}{3}$

Components of $\vec{a}$

$= \cos\alpha, \cos\beta, \cos r$

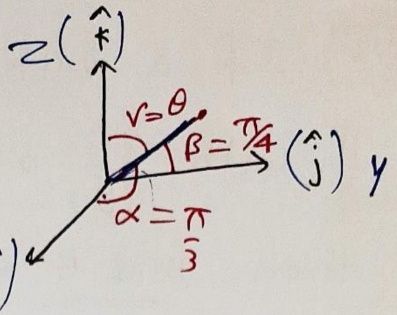
$= \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

(ii) $\alpha = \pi/3$

method $\beta = \pi/4$

$\gamma = \theta$

by vector product



Let unit vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i}$$

$$\Rightarrow \left| \underset{\substack{\downarrow\\1}}{|\vec{a}|}\,\underset{\substack{\downarrow\\1}}{|\hat{i}|} \sin\frac{\pi}{3} \cdot \underset{\substack{\downarrow\\1}}{\hat{n}} \right| = \left| -y\hat{k} + z\hat{j} \right|$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \sqrt{y^2 + z^2}$$

$$\Rightarrow \boxed{\frac{3}{4} = y^2 + z^2} \text{ — ①}$$

$\because \vec{a} \rightarrow$ unit vector

$|\vec{a}| = 1$

$$\underset{\substack{\downarrow\\ \frac{1}{4}}}{x^2} + \underset{\substack{\downarrow\\ \frac{1}{2}}}{y^2} + z^2 = 1 \text{ — ④}$$

Similarly, $\vec{a} \times \hat{j}$, $\vec{a} \times \hat{k}$

$$\boxed{\frac{1}{2} = x^2 + z^2} \text{ — ②} \qquad \boxed{\sin^2\theta = x^2 + y^2} \text{ — ③}$$

By eqⁿ ① & ④ : $x^2 = \frac{1}{4} \Rightarrow \boxed{x = \frac{1}{2}}$

By eqⁿ ② & ④ : $y^2 = \frac{1}{2} \Rightarrow \boxed{y = \frac{1}{\sqrt{2}}}$

Again by eqⁿ ④ : $z^2 = \frac{1}{4} \Rightarrow \boxed{z = \frac{1}{2}}$

Components

$$\sin^2\theta = x^2 + y^2$$

$$\sin^2\theta = \frac{1}{4} + \frac{1}{2}$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \sqrt{3}/2$$

$$\theta = \frac{\pi}{3}$$

$\boxed{Q.4}$ Show that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = 2(\vec{a}\times\vec{b})$

$$LHS = (\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$$

$$= \underbrace{\vec{a}\times\vec{a}}_{\vec{0}} + \underbrace{\vec{a}\times\vec{b}}_{✓} - \underbrace{\vec{b}\times\vec{a}}_{\text{change.}} - \underbrace{\vec{b}\times\vec{b}}_{\vec{0}}$$

$-\vec{b}\times\vec{a}$

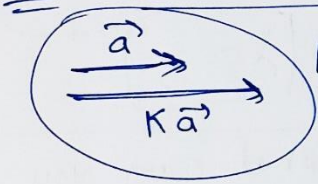
$= \vec{a}\times\vec{b}$

$$= (\vec{a}\times\vec{b} + \vec{a}\times\vec{b})$$

$$= 2(\vec{a}\times\vec{b}) = RHS.$$

$\boxed{Q.5}$ Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k}) = \vec{0}$

Ans. I - method,



$\boxed{\vec{a}\times(k\vec{a}) = \vec{0}}$

Collinear → Corresponding Components → Proportional

↑

Ratio Equal

$(2\hat{i}+6\hat{j}+27\hat{k})$ & $(\hat{i}+\lambda\hat{j}+\mu\hat{k})$ → Collinear.

$$\Rightarrow \frac{2}{1} = \frac{6}{\lambda} = \frac{27}{\mu}$$

$$\frac{2}{1} = \frac{6}{\lambda}$$

$$\Rightarrow \lambda = \frac{6}{2} = 3$$

$$\frac{2}{1} = \frac{27}{\mu}$$

$$\mu = \frac{27}{2}$$

II - method, $\underline{(2\hat{i}+6\hat{j}+27\hat{k})}\times\underline{(\hat{i}+\lambda\hat{j}+\mu\hat{k})}=\vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} \; \xrightarrow{} R_1 \quad = \vec{0} = 0.\hat{i}+0.\hat{j}+0.\hat{k}$$

$$\Rightarrow \underbrace{\hat{i}(6\mu-27\lambda)-\hat{j}(2\mu-27)+\hat{k}(2\lambda-6)}$$

$$= \underbrace{0.\hat{i}+0\hat{j}+0\hat{k}}$$

Comparison $\hat{j}$ $\quad 2\mu-27=0 \Rightarrow \mu=\frac{27}{2}$

$\hat{k}$ $\quad 2\lambda-6=0 \Rightarrow \boxed{\lambda=3}$

Q-6 Given that $\vec{a}\cdot\vec{b}=0$ & $\vec{a}\times\vec{b}=\vec{0}$. What can you conclude about the vectors $\vec{a}$ & $\vec{b}$?

$\vec{a}\cdot\vec{b}=0$

$\Rightarrow \boxed{|\vec{a}|\,|\vec{b}|\cos\theta=0}$

$\cancel{\theta=\frac{\pi}{2}}$

$\vec{a}\times\vec{b}=0$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta\cdot\hat{n}=\vec{0}$ } magnitude

$\Rightarrow \boxed{|\vec{a}|\,|\vec{b}|\sin\theta=0}$

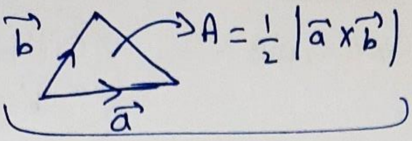
$\cancel{\theta\neq0}$

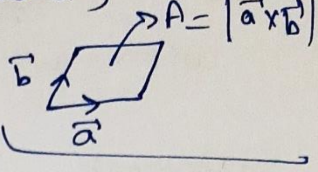
$\boxed{|\vec{a}|=0 \text{ or } |\vec{b}|=0}$ ←

[redacted] | Class 12 Maths | Vector Algebra

Exercise 10.4 → Cross Product (Vector Product)

(Q.7 to Q.12)





Q.7 Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

~~to~~ Ans. $\text{LHS} = \vec{a} \times (\vec{b} + \vec{c})$

$$= (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \left((b_1+c_1)\hat{i} + (b_2+c_2)\hat{j} + (b_3+c_3)\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix}$$

$\text{RHS} = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

$$= (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) + (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

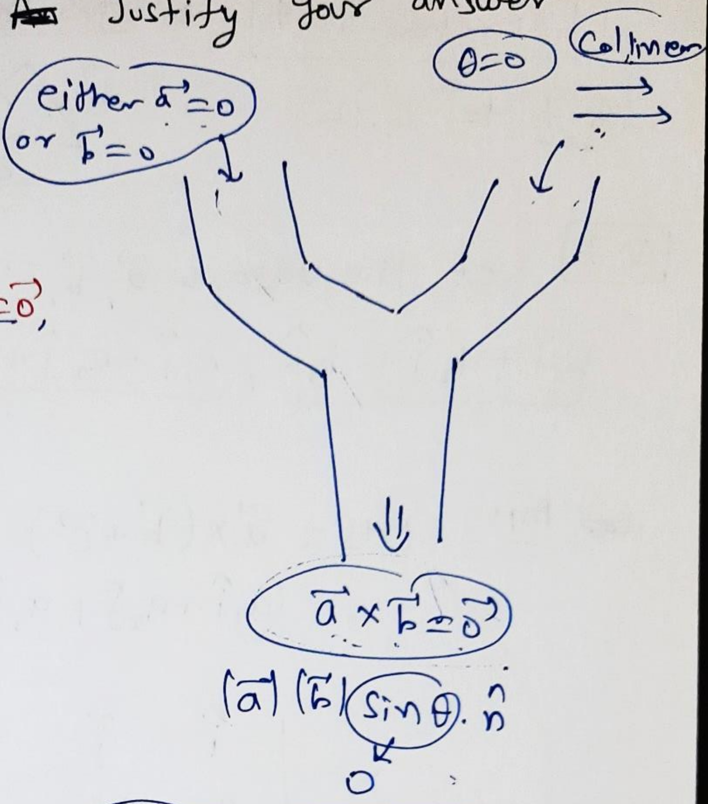
Property of Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ e & f \end{vmatrix} = \begin{vmatrix} a & b \\ c+e & d+f \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix} = \text{LHS}.$$

$\boxed{Q.8}$ If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\times\vec{b}=\vec{0}$.

Is the converse true? ~~A~~ Justify your answer with an example.



Statement

If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$,

then $\vec{a}\times\vec{b}=\vec{0}$.

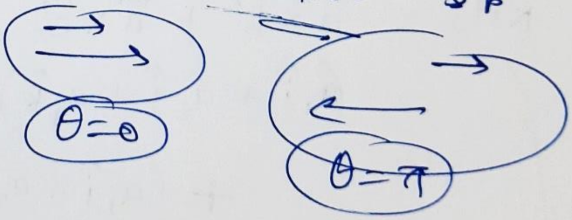
Converse Statement

If $\vec{a}\times\vec{b}=\vec{0}$

then either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$.

False.

$$\begin{tikzcd} & \text{either } \vec{a}=\vec{0} \text{ or } \vec{b}=\vec{0} \\ \vec{a}\times\vec{b}=\vec{0} \arrow[ur] \arrow[dr] & \\ & \text{Collinear vectors } \vec{a} \;\&\; \vec{b} \end{tikzcd}$$



$\vec{a}=\hat{i}\neq\vec{0}$

$\vec{b}=5\hat{i}\neq\vec{0}$

$\vec{a}\times\vec{0}=\vec{0}$

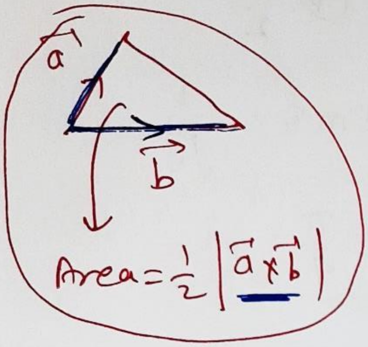
$$\vec{a}\times\vec{b}=\vec{0}$$

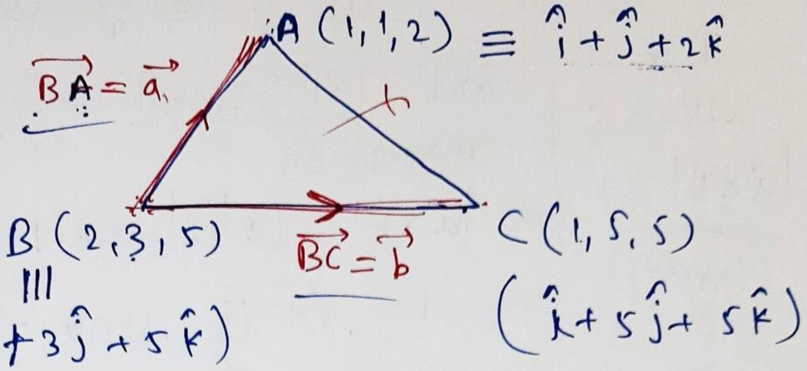
$$\Rightarrow (\hat{i})\times(5\hat{i})=\vec{0}$$

$$\Rightarrow 5(\vec{0})=\vec{0}$$

$$\vec{0}=\vec{0}$$

Q.9 Find the area of the triangle with vertices $A(1,1,2)$, $B(2,3,5)$ and $C(1,5,5)$.





$$\vec{a} = \overrightarrow{BA} = (-1)\hat{i} + (-2)\hat{j} + (-3)\hat{k}$$

$$\vec{a} = -\hat{i} - 2\hat{j} - 3\hat{k} \checkmark$$

$$\vec{b} = \overrightarrow{BC} = -\hat{i} + 2\hat{j} + 0\hat{k} \checkmark$$

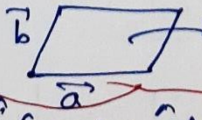
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ -1 & 2 & 0 \end{vmatrix} \xrightarrow{} R_1 = \hat{i}(6) - \hat{j}(-3) + \hat{k}(-4)$$

$$\boxed{\vec{a}\times\vec{b} = 6\hat{i} + 3\hat{j} - 4\hat{k}}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}\sqrt{36+9+16}$$

$$= \frac{1}{2}\sqrt{61} \checkmark$$

Q.10 Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ & $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.



$\rightarrow$ Area $= |\vec{a}\times\vec{b}|$

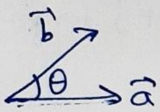
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(20) - \hat{j}(-5) + \hat{k}(-5)$$

$$\text{Area} = \sqrt{400+25+25}$$

$$A = \sqrt{450} = 15\sqrt{2}$$

Q.11 Let the vectors $\vec{a}$ & $\vec{b}$ be such that $|\vec{a}| = 3$ & $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ & $\vec{b}$ is —

(A) $\frac{\pi}{6}$ [✓ (B) $\frac{\pi}{4}$] (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$



$|\vec{a} \times \vec{b}| = 1$

Given

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$

magnitude

$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta \cdot 1$

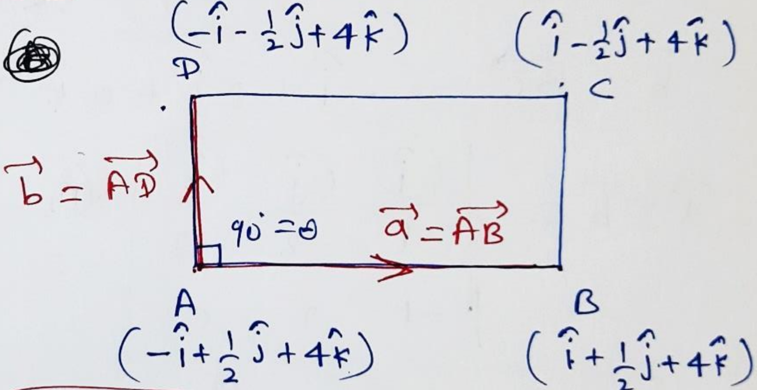
$\Rightarrow 1 = \cancel{3} \times \frac{\sqrt{2}}{\cancel{3}} \cdot \sin\theta$

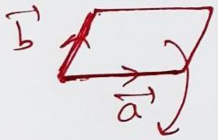
$\rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\boxed{\theta = \frac{\pi}{4}}$

Q.12 Area of a Rectangle having vertices A, B, C & D with position vectors $-\hat{i} + \frac{\hat{j}}{2} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$, $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$, respectively is —

(A) $\frac{1}{2}$ (B) 1 [✓ (C) 2] (D) 4



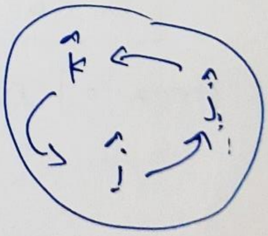


Area $= |\vec{a} \times \vec{b}|$

Final − initial

$\vec{a} = \vec{AB} = 2\hat{i} + 0\hat{j} + 0\hat{k} = 2\hat{i}$

$\vec{b} = \vec{AD} = 0 \cdot \hat{i} - \hat{j} + 0\hat{k} = -\hat{j}$



$\vec{a} \times \vec{b} = (2\hat{i}) \times (-\hat{j}) = -2(\hat{i} \times \hat{j})$

$\vec{a} \times \vec{b} = -2(\hat{k})$

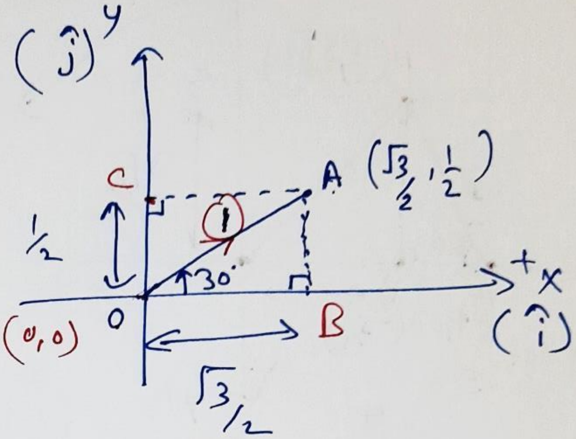
Area $= |\vec{a} \times \vec{b}| = |-2\hat{k}| = \sqrt{0 + 0 + 4} = 2$

Miscellaneous Exercise on Chapter - 10

Q.1 Write down a unit vector in XY-plane, making an angle 30° with the positive direction of x-axis.

Ans. $\vec{a} = \vec{OA} = ?$

$|\vec{a}| = 1 = |\vec{OA}| = OA$



ΔOAB $\cos 30° = \frac{OB}{OA}$

$\Rightarrow OB = OA \cdot \cos 30°$

$\Rightarrow OB = 1 \times \frac{\sqrt{3}}{2}$

$\boxed{OB = \frac{\sqrt{3}}{2}}$

$\sin 30° = \frac{AB}{OA} \xrightarrow{} 1 \Rightarrow AB = \sin 30°$

$\Rightarrow OC = \frac{1}{2} = AB$

$$\boxed{\vec{a} = \vec{OA} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}}$$

Q.2 Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

(Final - initial)

P → Q
(initial) (Final)

$$\boxed{\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$

Scalar Components $= (x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

$$\boxed{|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

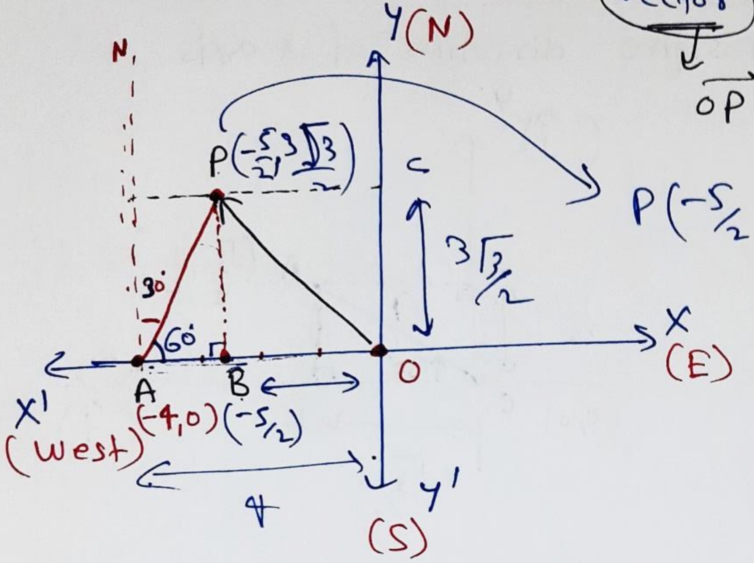
Q.3 A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's Displacement from her initial Point of Departure.

Vector

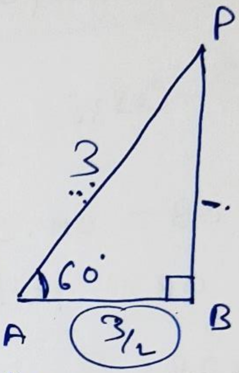
(0,0) O origin

$\vec{OP}$ = Final − initial

? (0,0)



$P\left(-\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$



$PB = \frac{3\sqrt{3}}{2}$

$\cos 60° = \frac{AB}{3} \Rightarrow AB = \frac{3}{2}$

$AB = \frac{3}{2}$

$\sin 60° = \frac{PB}{3} \Rightarrow PB = \frac{3\sqrt{3}}{2}$

X-coordinate of B = (X-coordinate of A) + AB

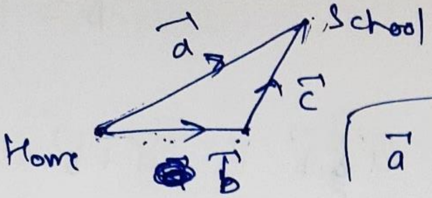
$= (-4) + 3/2 = -5/2$

$$\boxed{\vec{OP} = -\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}}$$

$(0,0)$ $\left(-\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$

Q.4 If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

No.



$\boxed{\vec{a} = \vec{b} + \vec{c}}$

$\vec{a} = \hat{i} + \hat{j}$

$|\vec{a}| = |\vec{b} + \vec{c}|$ ✓

$\vec{b} = \hat{i}.$

Here $\boxed{\vec{a} = \vec{b} + \vec{c}}$

$\vec{c} = \hat{j}.$

then $|\vec{a}| = |\vec{b}| + |\vec{c}|$

$|\hat{i} + \hat{j}| = |\hat{i}| + |\hat{j}|$

$\Rightarrow \sqrt{1+1} = 1 + 1$

$\Rightarrow \sqrt{2} = 2$ False ✓

Q.5 Find the value of x for which $\underbrace{x(\hat{i} + \hat{j} + \hat{k})}_{\vec{a}}$ is a unit vector.

$\vec{a} = x\hat{i} + x\hat{j} + x\hat{k}$

$|\vec{a}| = 1$

$\Rightarrow |x\hat{i} + x\hat{j} + x\hat{k}| = 1$

$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$

$\Rightarrow 3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3} \Rightarrow \boxed{x = \pm \frac{1}{\sqrt{3}}}$

$\boxed{Q.6}$ Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

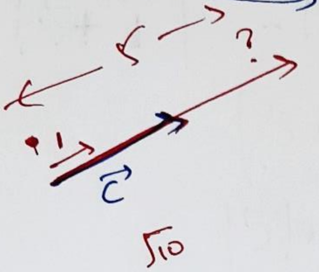
$$\underset{\boxed{\vec{a}+\vec{b}}}{\downarrow}$$

$\vec{c}$ = Resultant of $\vec{a}$ & $\vec{b}$ = $\vec{a} + \vec{b}$

$\vec{c} = (3\hat{i} + \hat{j})$

$|\vec{c}| = \sqrt{9+1}$

$= \sqrt{10}$



Here $\vec{c} = 3\hat{i} + \hat{j}$ (length $= \sqrt{10}$)

$\boxed{\text{unit vector}}$ $\hat{c} = \dfrac{\vec{c}}{|\vec{c}|} = \left(\dfrac{3\hat{i}+\hat{j}}{\sqrt{10}}\right)$ (length $= 1$)

Required vector $= 5\hat{c} = 5\left(\dfrac{3\hat{i}+\hat{j}}{\sqrt{10}}\right)$ (length $= 5$)

$= \cancel{5\left(\dfrac{3\hat{i}+\hat{j}}{\sqrt{2}\cdot\sqrt{5}}\right)} = \dfrac{5(3\hat{i}+\hat{j})}{\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}}$

$= \dfrac{\cancel{5}(3\hat{i}+\hat{j})\sqrt{10}}{\cancel{10}\ 2}$

$= \dfrac{3\sqrt{10}}{2}\hat{i} + \dfrac{\sqrt{10}}{2}\hat{j}$ ✓

$\boxed{Q.7}$ If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+3\hat{k}$ & $\vec{c} = \hat{i}-2\hat{j}+\hat{k}$, Find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$

$$2\vec{a}-\vec{b}+3\vec{c} = \cancel{2\hat{i}}+2\hat{j}+2\hat{k}-\cancel{2\hat{i}}+\hat{j}-\cancel{3\hat{k}} + 3\hat{i}-6\hat{j}+\cancel{3\hat{k}}$$

$$\vec{r} = 2\vec{a}-\vec{b}+3\vec{c} = 3\hat{i}-3\hat{j}+2\hat{k}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{9+9+4}} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}$$

Miscellaneous Exercise on Chapter - ⑩

Q.8 Show that the points A(1, −2, −8), B(5, 0, −2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.

Collinear Point

A, B, C → line

AB = $|\vec{AB}|$

BC = $|\vec{BC}|$

AC = $|\vec{AC}|$

A(1, −2, −8) / B(5, 0, −2) / C(11, 3, 7)

AC = AB + BC

Final − initial

$\vec{AB} = (4)\hat{i} + (2)\hat{j} + (6)\hat{k}$ → $|\vec{AB}| = \sqrt{16+4+36}$

$= \sqrt{56} = 2\sqrt{14}$

$\vec{BC} = (6)\hat{i} + (3)\hat{j} + (9)\hat{k}$ → $|\vec{BC}| = \sqrt{36+9+81} = 3\sqrt{14}$

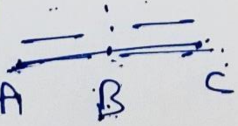
$\vec{AC} = 10\hat{i} + 5\hat{j} + 15\hat{k}$ → $|\vec{AC}| = \sqrt{100+25+225}$

$= \sqrt{350} = 5\sqrt{14}$

Here

$5\sqrt{14} = 2\sqrt{14} + 3\sqrt{14}$

$\boxed{AC = AB + BC}$

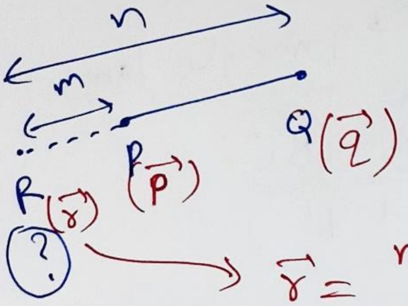


B Divides AC in Ratio AB : BC

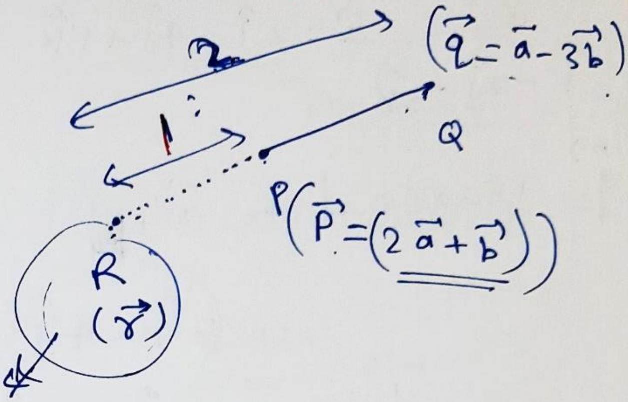
$= 2\cancel{\sqrt{14}} : 3\cancel{\sqrt{14}}$

$= 2 : 3$

Q.9 Find the position vector of a point R which divides the line joining two points P & Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1 : 2. Also show that P is the mid point of the line segment RQ.



$$\vec{r} = \frac{m\vec{q} - n\vec{p}}{m-n}$$

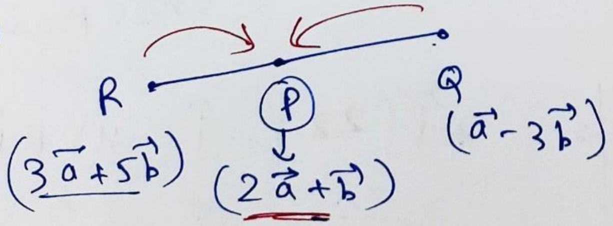


$$\vec{r} = \frac{1.\vec{q} - 2.\vec{P}}{1-2} = \frac{(\vec{a} - 3\vec{b}) - 2(2\vec{a}+\vec{b})}{-1}$$

$$\vec{r} = \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1}$$

$$\boxed{\vec{r} = 3\vec{a} + 5\vec{b}}$$ ← Position vector of 'R'

Also



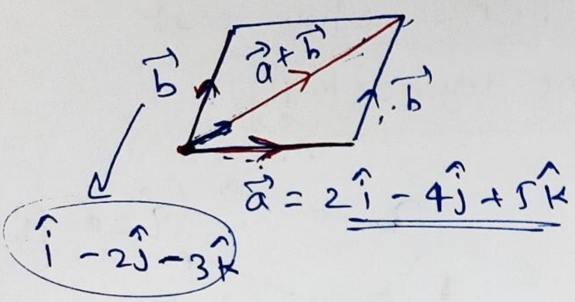
Mid Point of R & Q $= \dfrac{\vec{r} + \vec{q}}{2} = \dfrac{(3\vec{a}+5\vec{b}) + (\vec{a}-3\vec{b})}{2}$

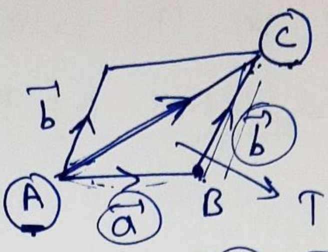
$$= 2\vec{a} + \vec{b}$$

$$= \vec{P}$$

$\boxed{Q.10}$ The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its Area.

Cross Product.





Triangle law

$$\vec{AC} = \vec{AB} + \vec{BC}$$

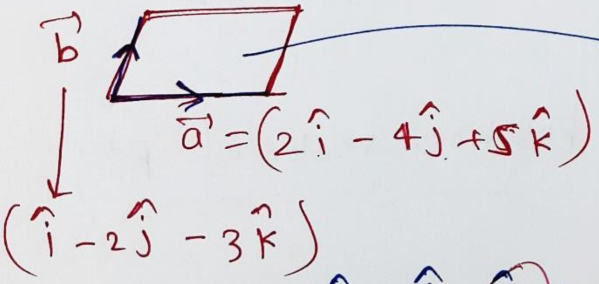
$$\vec{a} + \vec{b}$$

$$\vec{d} = \text{Diagonal} = \vec{a}+\vec{b}$$

$$= (2\hat{i}-4\hat{j}+5\hat{k}) + (\hat{i}-2\hat{j}-3\hat{k})$$

$$\vec{d} = 3\hat{i}-6\hat{j}+2\hat{k}$$

$$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}} = \left(\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}\right)$$



$$\text{Area} = |\vec{a}\times\vec{b}|$$

Cross Product

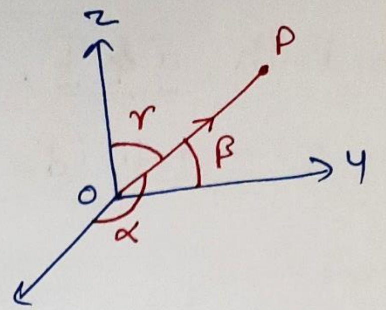
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} = +\hat{i}(22) - \hat{j}(-11) + \hat{k}(0)$$

$$= (22\hat{i} + 11\hat{j} + 0\hat{k})$$

$$\text{Area} = |\vec{a}\times\vec{b}| = \sqrt{484+121+0} = \sqrt{484+121+0}$$

$$= \sqrt{605} = \sqrt{5\times121} = 11\sqrt{5}$$

Q.11 Show that the direction cosines of a vector equally inclined to the axes OX, OY, OZ are $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.



$\alpha, \beta, \gamma \rightarrow$ Direction Angles

$\cos\alpha, \cos\beta, \cos\gamma \rightarrow$ Direction Cosines → Property

$$\boxed{\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1}$$

equally inclined → (angles → equal,)
(OX, OY, OZ)

$$\alpha=\beta=\gamma$$

$$\Rightarrow \boxed{\cos\alpha=\cos\beta=\cos\gamma} \text{ ATQ.}$$

$$\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$\Rightarrow \cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1 \Rightarrow 3\cos^2\alpha=1$$

$$\Rightarrow \cos^2\alpha=\frac{1}{3} \Rightarrow \cos\alpha=\pm\frac{1}{\sqrt{3}}=\cos\beta=\cos\gamma$$

D.C. → $\cos\alpha, \cos\beta, \cos\gamma$

$$= \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$$

$\boxed{\text{Q.12}}$ Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ & $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ & $\vec{b}$, and $\vec{c}\cdot\vec{d} = 15$.

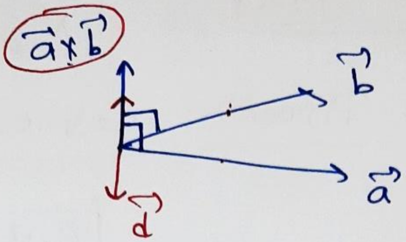
Ans. I-method $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$ → Cond^n.

$\vec{a}\cdot\vec{d} = 0$ — ①

$\vec{b}\cdot\vec{d} = 0$ — ②

$\vec{c}\cdot\vec{d} = 15$ — ③

II-method



$(\vec{a}\times\vec{b}) \perp \vec{a}$

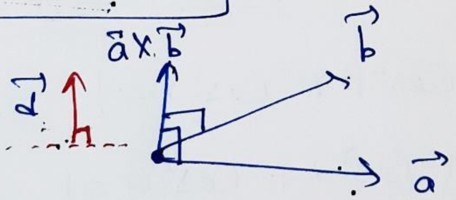
$(\vec{a}\times\vec{b}) \perp \vec{b}$

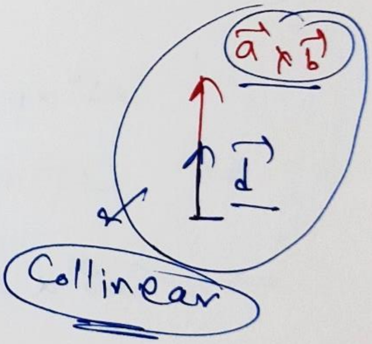
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = \hat{i}(32) - \hat{j}(1) + \hat{k}(-14)$$

(Cross product)

$$= \underbrace{32\hat{i} - \hat{j} - 14\hat{k}}$$

$$\boxed{\vec{a}\times\vec{b} = 32\hat{i} - \hat{j} - 14\hat{k}}$$





Here $\vec{d}$ & $(\vec{a}\times\vec{b})$ are collinear vectors.

$\vec{d} = \lambda(\vec{a}\times\vec{b})$

$$\boxed{\vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})}$$

Also $\vec{c}\cdot\vec{d} = 15$

$\Rightarrow (2\hat{i} - \hat{j} + 4\hat{k})\cdot\lambda(32\hat{i} - \hat{j} - 14\hat{k}) = 15$

$\Rightarrow \lambda(64 + 1 - 56) = 15$

$\Rightarrow \lambda(9) = 15 \Rightarrow \boxed{\lambda = 15/9 = \frac{5}{3}}$

Answer

$\vec{d} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})$

$\vec{d} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$

Miscellaneous Exercise on Chapter - ⑩

Q.13 The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector ($\hat{c}$) along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$, is equal to one. Find the value of λ.

Ans. Sum of vectors $(2\hat{i}+4\hat{j}-5\hat{k})$ & $(\lambda\hat{i}+2\hat{j}+3\hat{k})$

$$\vec{c} = (2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+36+4}} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+\lambda^2+4\lambda+40}}$$

$$\hat{c} = \left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right)$$

ATQ. Scalar Product (Dot) of $(\hat{i}+\hat{j}+\hat{k})$ with $\hat{c} = 1$

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}).(\hat{c}) = 1$$

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\cdot\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right) = 1$$

$$\Rightarrow (\cancel{2}+\lambda)+6-\cancel{2} = \sqrt{\lambda^2+4\lambda+44}$$

$$\Rightarrow (\lambda+6)^2 = (\sqrt{\lambda^2+4\lambda+44})^2$$

$$\Rightarrow \cancel{\lambda^2}+12\lambda+36 = \cancel{\lambda^2}+4\lambda+44$$

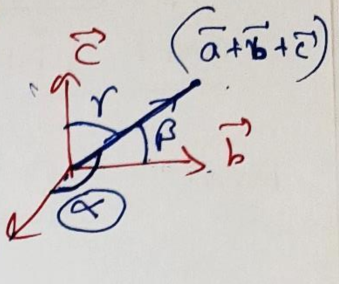
$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \boxed{\lambda = 1}$$

Q.14 If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}, \vec{b}$ & $\vec{c}$.

Ans Given: $\boxed{\vec{a}\cdot\vec{b}=0}$, $\boxed{\vec{b}\cdot\vec{c}=0}$, $\boxed{\vec{a}\cdot\vec{c}=0}$

$|\vec{a}|=|\vec{b}|=(\vec{c})=K$ (let)



let α, β & γ be the angles between $(\vec{a}+\vec{b}+\vec{c})$ and $\vec{a}, \vec{b}, \vec{c}$ respectively.

To Prove: (equally inclined) → angle same

$\boxed{\alpha=\beta=\gamma}$ ←

~~Sol~~ Proof:

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{a}|\cos\alpha$

$\Rightarrow \vec{a}\cdot\vec{a} + \underbrace{\vec{b}\cdot\vec{a}}_{0} + \underbrace{\vec{c}\cdot\vec{a}}_{0} = |\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{a}|\cdot\cos\alpha$

$\Rightarrow |\vec{a}|^{\cancel{2}} = |\vec{a}+\vec{b}+\vec{c}|\cdot\cancel{|\vec{a}|}\cdot\cos\alpha$

$\Rightarrow |\vec{a}| = |\vec{a}+\vec{b}+\vec{c}|\cos\alpha$

$\Rightarrow \cos\alpha = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$

$\Rightarrow \boxed{\cos\alpha = \frac{K}{|\vec{a}+\vec{b}+\vec{c}|}}$ — ①

Similarly,

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}$

$\boxed{\cos\beta = \frac{K}{|\vec{a}+\vec{b}+\vec{c}|}}$ — ②

$\boxed{\cos\gamma = \frac{K}{|\vec{a}+\vec{b}+\vec{c}|}}$ — ③

By eqⁿ ①, ② & ③ :→

$\boxed{\cos\alpha=\cos\beta=\cos\gamma} \Rightarrow \boxed{\alpha=\beta=\gamma}$

Q.15 Prove that $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2+|\vec{b}|^2$, if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0}$, $\vec{b}\neq\vec{0}$.

Ans. $(\vec{a}+\vec{b}).(\vec{a}.+\vec{b}) = |\vec{a}|^2+|\vec{b}|^2 \Longleftrightarrow \vec{a}\perp\vec{b}$

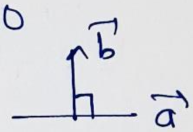
Part (I) Let $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2+|\vec{b}|^2$

$$\Rightarrow \vec{a}\cdot\vec{a} + \underbrace{\vec{a}.\vec{b}} + \underbrace{\vec{b}\cdot\vec{a}} + \vec{b}\cdot\vec{b} = |\vec{a}|^2+|\vec{b}|^2$$

$$\Rightarrow \cancel{|\vec{a}|^2} + 2(\vec{a}\cdot\vec{b}) + \cancel{|\vec{b}|^2} = \cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}$$

$$\Rightarrow 2(\vec{a}\cdot\vec{b}) = 0$$

$$\Rightarrow \boxed{\vec{a}\cdot\vec{b}=0}$$



Part (II) Let $\vec{a}\perp\vec{b} \Rightarrow \boxed{\vec{a}.\vec{b}=0}$

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

$$= |\vec{a}|^2 + 0 + 0 \quad |\vec{b}|^2$$

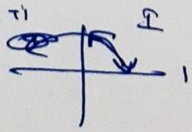
$$= |\vec{a}|^2+|\vec{b}|^2$$

Q.16 If θ is the angle between two vectors $\vec{a}$ & $\vec{b}$, then $\vec{a}\cdot\vec{b} \geq 0$ only when – ~~(A) $0<\theta<\frac{\pi}{2}$~~ (B) $0\leq\theta\leq\frac{\pi}{2}$

~~(C) $0<\theta<\pi$~~ ~~(D) $0\leq\theta\leq\pi$~~

$\Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta \geq 0$

$\Rightarrow \boxed{\cos\theta \geq 0}$



$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

$\cos\theta > 0$

+ve

in I- quadrant $\rightarrow \theta \in [0, \frac{\pi}{2})$

Q.17 Let $\vec{a}$ & $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{2}$ [✓ (D) $\theta = \frac{2\pi}{3}$]

$\vec{a}\cdot\vec{a} = |\vec{a}|^2$

$|\vec{a}| = |\vec{b}| = 1$, $|\vec{a}+\vec{b}| = 1$

$\Rightarrow |\vec{a}+\vec{b}|^2 = 1$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1$

$\Rightarrow |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + |\vec{b}|^2 = 1$

$\Rightarrow 1 + 2(\vec{a}\cdot\vec{b}) + \cancel{1} = \cancel{1}$

$\Rightarrow \vec{a}\cdot\vec{b} = -\frac{1}{2}$

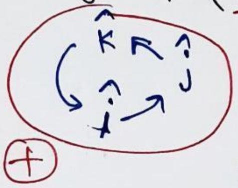
$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = -\frac{1}{2}$

$\Rightarrow \boxed{\cos\theta = -\frac{1}{2}}$

$\theta = 120° = \frac{2\pi}{3}$

Q.18. The value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{k}\cdot(\hat{i}\times\hat{j})$ is →

(A) 0 (B) −1 [✓ (C) 1] (D) 3



$= \hat{i}\cdot(\hat{i}) + \hat{j}\cdot(-\hat{j}) + \hat{k}(\hat{k})$

$= \cancel{1} - \cancel{1} + 1 = 1$

Q.19 If θ is the angle between any two vectors $\vec{a}$ & $\vec{b}$, then $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, when θ is equal to—

(A) 0 [✓ (B) $\frac{\pi}{4}$] (C) $\frac{\pi}{2}$ (D) π

$|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$\Rightarrow \big|\,|\vec{a}||\vec{b}|\cos\theta\,\big| = \big\|\,|\vec{a}||\vec{b}|\sin\theta\,.\hat{n}\,\big\|$

$\Rightarrow \cancel{|\vec{a}|}\cancel{|\vec{b}|}\cos\theta = \cancel{|\vec{a}|}\cancel{|\vec{b}|}\sin\theta\,.\,|\hat{n}| \rightarrow 1$

$\Rightarrow 1\ \cos\theta = \sin\theta$

$\Rightarrow 1 = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \boxed{\tan\theta = 1}$ $\theta = 45° = \frac{\pi}{4}$